所別：經濟學系碩士班 科目：統計學

1. (5 分) 給定 $\mathbb{P}(B) = 0.9$, $\mathbb{P}(A|B^c) = 0.7$, $\mathbb{P}(A|B) = 0.2$, 請求出 $\mathbb{P}(A)$ and $\mathbb{P}(B|A)$。

2. (5 分) 假設台積電股價報酬率 X 爲一隨機變數, 若已知台積電股價報酬率期望值爲 2%, 且 $X \le -2\%$ 的機率爲 0.3, $X \ge 6\%$ 的機率爲 0.1, 則購買台積電股票的風險 (以標準差衡量) 不可能小於多少?

3. (10 分) 請舉例說明「不偏 (unbiased) 估計式未必是一致性 (consistent) 估計式, 且一致性估計式也不一定是不偏估計式。」

4. (10 分) X, Y 爲隨機變數, 且 $\mathbb{E}(X) = 0$, $\text{var}(X) = 4$, $\mathbb{E}(Y|X) = 2 - 3X$, 請計算 $\mathbb{E}(XY)$, $\text{cov}(X,Y)$, 又 X 與 Y 是否獨立?

5. (20 分) 我們從台灣大學 (T) 和中央大學 (C) 經濟系的畢業生中各隨機抽出 9 名, 並得到其年薪的樣本平均數和樣本變異數資料如下:

   $\bar{Y}_T = 19$ (單位: 十萬)　$S_T^2 = 4$

   $\bar{Y}_C = 21$　$S_C^2 = 5$

   假設這些薪資爲常態分配, 在 5% 的顯著水準, 請說明檢定以下兩種假設的步驟。

   (a) 兩個學校經濟系畢業生的薪資變異數相等。

   (b) 兩個學校經濟系畢業生的薪資均數相等。

6. By using a cross-sectional data of sample size 100 to estimate the regression model, $M_i = \beta_1 + \beta_2 W_{2i} + \beta_3 W_{3i} + \beta_4 X_i + \varepsilon_i$. The following empirical result is obtained:

$$\hat{M}_i = \underset{(0.55)}{1.23} - \underset{(0.25)}{0.35}W_{2i} + \underset{(0.95)}{1.51}W_{3i} + \underset{(0.23)}{0.55}X_i$$

, where the standard errors are in the parentheses and $\bar{R}^2 = 0.85$.

According to the information below, identify the statements which are correct and explain why. Modify the statement if it is false.

Statement A: The model specification is appropriate since $\bar{R}^2$ is quite high. (7%)

Statement B: $E(M_i \mid W_{2i}, W_{3i}, X_i) = \hat{\beta}_1 + \hat{\beta}_2 W_{2i} + \hat{\beta}_3 W_{3i} + \hat{\beta}_4 X_i$. (7%)

注意：背面有試題

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Statement C: According to the following hypothesis test with the significance level $\alpha = 0.05$, W_{2i} is not an important explanatory variable for the model. (7%)

$$H_0 : the\ coefficient\ of\ W_{2i} = 0$$
$$H_1 : the\ coefficient\ of\ W_{2i} \neq 0$$
$$t = 0.35 / 0.25 = 0.14$$
$$|t| < t_{0.05}(96) \cong 2.33$$

Statement D: If all the variables in the model are standardized, then the coefficient estimates represent the elasticities. (7%)

Statement E: Suppose a researcher wants to test if the marginal effects of W_{2i} and W_{3i} are the same. Then the following hypothesis test can be conducted where an F test can be used. (7%)

$$H_0 : \hat{\beta}_2 = \hat{\beta}_3$$
$$H_1 : \hat{\beta}_2 \neq \hat{\beta}_3$$

Statement F: Suppose Cov(W_{2i},W_{3i})=0.88. The estimators of βs are biased, while the biasedness depends on the magnitude of the covariance between W_{2i} and W_{3i} . (7%)

Statement G: If $Var(\varepsilon_i \mid X_i\ is\ large) > Var(\varepsilon_i \mid X_i\ is\ small)$, then ε_i and ε_j are correlated in the sense that $Cov(\varepsilon_i, \varepsilon_j) \neq 0 \quad \forall i \neq j$, and the Gauss-Markov Theorem couldn't be applied. (8%)