國立中央大學100學年度碩士班考試入學試題卷

所別：數學系碩士班 甲組(一般生)　科目：高等微積分　共 1 頁 第 1 頁
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本科考試禁用計算器

*請在試卷答案卷（卡）內作答

參考用

Let $\mathbb{N}$ be the natural numbers and $\mathbb{R}$ be the real numbers.

1. Prove or disprove the following statements:
   (1) Let (M, d) be a metric space. Then the union of an arbitrary family of closed subsets is closed. (10%)
   (2) Let (M, d) be a metric space. Then every bounded and closed set is compact. (10%)
   (3) Let $\{p_n\}_{n\in\mathbb{N}}$ be a uniformly convergent sequence of polyniomials on $[0, 2]$ and $f = \lim_{n\to\infty} p_n$. Then f is differentiable. (10%)
   (4) Suppose that $f : [0, 1] \to \mathbb{R}$. Then f is Riemann integrable if and only if $|f|$ Riemann integrable. (10%)

2. If $f : \mathbb{R} \to \mathbb{R}$ is a continuous function and $f\big(f(x)\big) = x$ for all $x \in \mathbb{R}$. Show that there exists a $\xi \in \mathbb{R}$ such that $f(\xi) = \xi$. (12%)

3. Let $f(x) = \sqrt{x}$ on $[0, \infty)$. Prove or disprove f is uniformly continuous. (12%)

4. Test the following series for convergence. (a) $\sum_{n=0}^{\infty} \frac{\sqrt{n+3}}{n^2-3n+1}$ (6%);
(b) $\sum_{k=1}^{\infty} \frac{\ln(k+2)-\ln k}{\tan^{-1}(2/k)}$. (6%)

5. Suppose that $\{a_n\}_{n\in\mathbb{N}}$ is a bounded sequence of real numbers. Show that $f(x) = \sum_{n=1}^{\infty} \frac{a_n}{n!} x^n$ is a continuous function on $\mathbb{R}$. (12%)

6. Let $L : \mathbb{R}^n \to \mathbb{R}^n$ be a linear map. Prove that L is continuous and differentiable on $\mathbb{R}^n$ by definition. (12%)