

國立中央大學104學年度碩士班考試入學試題

所別：數學系碩士班 一般組(一般生) 科目：高等微積分 共 / 頁 第 / 頁
數學系碩士班 一般組(在職生)

本科考試禁用計算器

*請在答案卷(卡)內作答



1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined on \mathbb{R} . Please use the $\epsilon - \delta$ language to define what it means for the function $f(x)$ to be continuous at $x = 1$. (10%)
2. Please use the $\epsilon - \delta$ language to prove that $\lim_{x \rightarrow 1} \frac{1}{x} = 1$. (10%)
3. Let a_1, a_2, a_3, \dots be a sequence of positive numbers. Please give a definition for what the following limit means:

$$\lim_{n \rightarrow \infty} a_n = 2015.$$

(10%)

4. Let $f_n(x) : [0, 1] \rightarrow \mathbb{R}$, $n = 0, 1, 2, \dots$, be a sequence of functions. Please give a definition for what it means for $\{f_n(x)\}_{n \geq 1}$ to be uniformly convergent to $f_0(x)$. (10%)
5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined on \mathbb{R} . Please give a definition for $f(x)$ to be uniformly continuous. (10%)
6. Please find the antiderivative: $\int \ln(x) dx =$ (10%)
7. Please find the limit: $\lim_{x \rightarrow 0^+} x^x =$ (10%)
8. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous, then show that

$$\int_0^x \left[\int_0^u f(t) dt \right] du = \int_0^x f(u)(x-u) du.$$

(10%)

9. Give an example of a function $f : [0, 1] \rightarrow \mathbb{R}$ such that f is not Riemann integrable. Please justify your answer. (20%)