

國立中央大學 106 學年度碩士班考試入學試題

所別： 數學系 碩士班 數學組(一般生)

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數學系 碩士班 數學組(在職生)

科目： 高等微積分

本科考試禁用計算器

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ADVANCED CALCULUS MASTER ENTRANCE EXAM 2017/02/07

Do the following problems and write your arguments as detail as possible

1. (10%) Let $f(x)$ be a continuous function on $[0, 1]$ that satisfies $\int_0^x f(t)dt = \int_x^1 f(t)dt$ for all $x \in [0, 1]$. Show that $f(x) = 0$ for all x .
2. (10%) Suppose $f(x, y)$ is a differentiable function on a connected convex open set S and $\partial_y f(x, y) = 0$ for all $(x, y) \in S$. Show that given any two points $a = (a_1, a_2), b = (b_1, b_2) \in S$ as long as $a_1 = b_1$ then $f(a) = f(b)$.
3. (10%) Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{(n+1)2^{2n}}$. Find all the values of x such that the series converges.
4. (10%) Find a vector field $F = (F_1(x, y, z), F_2(x, y, z), F_3(x, y, z))$ so that $\text{div} F = g$, where $g(x, y, z) = x^2y + xyz$.
5. (10%) Find the volume of the ice cream cone T bounded below by the cone $z = 2\sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 1$.
6. (10%) Find the limit value $\lim_{x \rightarrow 0^+} \sqrt{x}e^{\sin(1/x)}$ and prove your result.
7. (10%) Let $f(x, y) = \sin(x^2 + y)e^{xy}$ and $a = (0, 0)$. Find the third order Taylor polynomial of f at a .
8. (10%) Is it possible to find a function $f(x)$ on $[0, 1]$ so that f is not Riemann integrable, but $|f|$ is Riemann integrable? Prove or disprove your result.
9. (10%) Let C be the unit circle $x^2 + y^2 = 1$, oriented counterclockwise. Compute the line integral $\int_C [\sqrt{1+x} - ye^{xy} + y^2]dx + [3x^2 - xe^{xy} + \log(1+y^2)]dy$.
10. (10%) Let $F(x) = \int_0^{\infty} e^{-t^2} \cos(xt)dt$. (a) Show that $F'(x) = -\frac{1}{2}xF(x)$. (b) Given $\int_0^{\infty} e^{-t^2} = \frac{\sqrt{\pi}}{2}$. Show that $F(x) = \frac{\sqrt{\pi}}{2}e^{-\frac{x^2}{4}}$.