國立中央大學 109 學年度碩士班考試入學試題

所別： 數學系 碩士班 應用數學組(一般生)
數學系 碩士班 應用數學組(在職生)

科目： 微積分

本科考試禁用計算器 計算題需計算過程，無計算過程者不予計分 ＊請在答案卷(卡)內作答

Problem 1. (10%) Find $\lim_{x\to\infty} x\left[\left(1+\frac{1}{x}\right)^x - e\right]$.

Problem 2. (10%) Show that $\frac{d}{dx}\int_{[\log_\pi(e+x^2)]^2}^{(\log_\pi 2)^2} \pi^{\sqrt{u}}\,du = -\frac{2\ln(e+x^2)}{(\ln\pi)^2}$.

Problem 3. (10%) Evaluate $\int_0^1 x^2(\ln x)^3\,dx$.

Problem 4. (10%) Find the indefinite integral $\int \frac{dx}{x^4(x^2+1)}$.

Problem 5. (10%) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{3+\cos^2 x}$.

Problem 6. (10%) Find a positive integer n such that $\left|e - \sum_{k=0}^{n} \frac{1}{k!}\right| < 10^{-8}$. Explain your answer.

Problem 7. (10%) Find the relative extrema of the function $f(x,y) = (1-2x^2+2y^2)e^{1-x^2-y^2}$. Use the second derivative test when applicable.

Problem 8. (10%) Find the maximum and minimum value of the n-variable function $x_1+x_2+\cdots+x_n$ subject to the constraint $x_1^2+x_2^2+\cdots+x_n^2=1$ by the method of Lagrange multipliers.

Problem 9. (10%) Evaluate the double integral

$$\iint_R (x-y)\cos\frac{y+x}{y-x}\,dA,$$

where R is the trapezoidal region with vertices $(1,0)$, $(2,0)$, $(0,-2)$ and $(0,-1)$.

Problem 10. (10%) Find the area of the region bounded by the graphs of the polar equations $r = 2\sin\theta$, $\theta = \frac{\pi}{4}$ and $\theta = \frac{\pi}{2}$.