參考用

1. Let G be a finite simple graph. The set of edges of G forms a line graph L(G) (two vertices of L(G) are adjacent if and only if they share a common vertex in G). Let K_5 be a complete graph with 5 vertices; P be the complement of the line graph L(K_5).

a. Find the connectivity of P and prove it. (10 %)

b. What is the diameter of P? (10 %)

c. Prove or disprove that P is Hamiltonian. (10 %)

d. Find the covering number, independent number and chromatic number of P and prove it. (10 %)

e. What is the complexity (number of labelled spanning trees) of P ? (10 %)

f. Prove or disprove that P is a planar graph. (10 %)

2. Prove or disprove that there is an Hadamard matrix of order 4m (a square (1,-1) matrix H such that $H \cdot H^t = n \cdot I$) if and only if there is a square (0,1) matrix A of order 4m-1 such that $A \cdot A^t = m \cdot I + (m-1) \cdot J$, where I is a identity matrix, J is all 1 matrix. (10 %)

3. Let $(\Sigma_{i=0}^{\infty} x^i)^r = \Sigma_{k=0}^{\infty} a_k \cdot x^k$ for a fixed constant r . Find a_k . (10 %)

4. Find the solution to $S_n = 2 \cdot S_{n-1} + 2^n$, $S_0 = 1$. (10 %)

5. Let G be a finite group and let H be a subgroup of G. Prove or disprove that there exist elements h_1, h_2 , ... h_n in G such that $h_1 \cdot H$, $h_2 \cdot H$, ... , $h_n \cdot H$ are set of all left cosets and $H \cdot h_1$, $H \cdot h_2$, ... ,$H \cdot h_n$ are set of all right cosets of G. (10 %)