離散數學

每題 25 分

1. A diagonal of an n by n matrix is a set of n entries no two of which belong to the same row or the same column. The weight of a diagonal is the sum of the entries in it. Find a minimum-weight diagonal in the following matrix:

$$\begin{pmatrix} 5 & 6 & 9 & 11 & 12 \\ 8 & 7 & 6 & 8 & 5 \\ 9 & 6 & 13 & 10 & 7 \\ 7 & 7 & 10 & 9 & 7 \\ 5 & 6 & 7 & 9 & 8 \end{pmatrix}.$$

2. The ramsey number is the smallest integer r(k,s) such that every graph contains either a clique of k vertices or an independent set of s vertices if the size of the vertex set is at least r(k,s).

a. Prove that $r(k,s) \leq r(k,s-1) + r(k-1,s)$.

b. Find r(3,4).

3. A subset M of the edge set of a graph G is a perfect matching if M define a subgraph of G such that every vertex has degree 1. Prove or disprove that a k-regular bipartite graph has a perfect matching.

4. The odd graph O_n is defined as follows:

The vertex set is the set of all k-1 subsets of a set of cardinality 2k-1, two vertices are adjacent if and only if the corresponding subsets are disjoint.

a. Find the connectivity, chromatic number and independent number of O_3.

b. Let E be the incidence matrix of O_3. What is the rank of E over the binary field (field contains only two elements 0,1) ? What is the meaning of this rank in graph theory?