

國立中央大學八十五學年度碩士班研究生入學試題卷

所別: 數學研究所 不分組 科目: 複變函數論 共 1 頁 第 1 頁

(20 pts) 1. Find a linear fractional transformation $S(z)$ which maps the unit disk onto the right half plane and $S(0) = 2$.

(20 pts) 2. If $f(z)$ is analytic in $|z| < 1 + \varepsilon$ for some $\varepsilon > 0$ and $|f(z)| \leq M$ in $|z| \leq 1$, show that $\left| \frac{df}{dz} \right| \leq 4M$ for $|z| \leq \frac{1}{2}$.

(20 pts) 3. Show that all the zeros of $2z^5 + 8z - 11$ lie in $1 < |z| < 2$.

(20 pts) 4. Compute the integral

$$\int_{\gamma} \frac{dz}{\sin \frac{1}{z}}$$

where γ is the circle $|z| = 1$ traversed once in a positive sense.

(20 pts) 5. Find the Laurent series of the function $f(z) = \frac{z}{z^2 - 3z + 2}$ valid in

(1) $1 < |z| < 2$

(2) $|z - 1| > 1$.