

國立中央大學八十六學年度碩士班研究生入學試題卷

所別： 數學研究所 不分組 科目： 高等微積分 共 1 頁 第 1 頁

1. Let D be a bounded subset of the set of real numbers R . Show that $f: D \rightarrow R$ is bounded on D if and only if for each $x \in R$, there is a neighborhood N_x of x such that f is bounded on $D \cap N_x$. (20%)

2. Let $f(x) = 0$ and $f_n = nx e^{-nx^2}$, $n = 1, 2, 3, \dots$. Show that the sequence $\{f_n\}$ converges to f pointwise but not uniformly on $[0, 1]$. (20%)

3. Let $D = \{(x, y) : 1 < x^2 + y^2 < 4\}$. Assume $f: D \rightarrow R$ satisfies $f_x(x, y) = 0$ and $f_y(x, y) = 0$ on D . Show that f is constant on D . (10%)

4. Let $D = \{(x, y) : x > 0, y > 0, 1 \leq xy \leq 3, 1 \leq x^2 - y^2 \leq 7\}$. Evaluate $\iint_D x^2 + y^2 dx dy$. (10%)

5. Show that $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$ for all x in $(-1, 1]$. (20%)

6. Suppose $f: R^2 \rightarrow R$ and $g: R^2 \rightarrow R$ are both continuous on R^2 , differentiable at $(0, 0)$, and $f_x(0, 0) = g_y(0, 0)$. Show that for each $\epsilon > 0$, there is a $\delta > 0$ such that $\left| \oint_{x^2+y^2=r^2} f(x, y) dy + g(x, y) dx \right| < \epsilon r^2$ whenever $0 < r < \delta$. (20%)

