

# 國立中央大學八十九學年度碩士班研究生入學試題卷

70 所別: \_\_\_\_\_ 數學系 不分組 科目: \_\_\_\_\_ 微分方程 共 | 頁 第 | 頁

1. (20%) (a) Solve the initial value problem

$$\begin{cases} \frac{dy}{dt} = r_0(1 - \frac{y}{K_0})y, & t > 0 \\ y(0) = y_0 > 0, \end{cases} \quad (1)$$

where  $r_0$  and  $K_0$  are two positive constants.

- (b) Let  $y$  be the solution of equation (1). Prove:  $\lim_{t \rightarrow \infty} y(t) = K_0$ .

2. (20%) (a) Find the general solution of the following equation

$$X'(t) = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 3 \\ 2 & 3 & 0 \end{pmatrix} X(t), \text{ where } X(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}. \quad (2)$$

- (b) Let

$$S = \left\{ \begin{pmatrix} u \\ v \\ w \end{pmatrix} \in \mathbb{R}^3 \mid X(t) \text{ is a solution of Eq. (2) with } X(0) = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \right. \\ \left. \text{such that } \lim_{t \rightarrow \infty} X(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}.$$

Find the set  $S$  and prove that  $S$  is a linear vector space of dimension 2.

3. (21%) Solve the following differential equations respectively.

(a)  $(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$

(b)  $y'' - 3y' - 4y = -8e^t \cos 2t$

- (c) Given that  $y_1 = x + 1$  is a solution of

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1) \frac{dy}{dx} + 3y = 0, \quad (3)$$

find a linearly independent solution  $y_2$  of Eq. (3). Write the general solution.

4. (21%) Find a series solution in powers of  $x$  of Airy's equation

$$y'' - xy = 0, \quad -\infty < x < \infty.$$

5. (18%)(a) Solve the initial value problem

$$\begin{cases} y''(t) + 5y'(t) + 6y(t) = 0, & t > 0 \\ y(0) = 2, y'(0) = \beta > 0. \end{cases} \quad (4)$$

- (b) Determine the coordinates  $t_0$  and  $y_0$  of the maximum point of the solution as functions of  $\beta$ .

- (c) Determine the behavior of  $t_0$  and  $y_0$  as  $\beta \rightarrow \infty$ .

參考用