# 國立中央大學九十一學年度碩士班研究生入學試題卷

所別: 數學系 不分組 科目: 微分方程 共一頁 第一頁

1.(10%) Solve the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0.$$

2.(20%) Find a power series solution of the initial-value problem

$$(x^2 - 1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0.$$

$$y(0) = 4, \quad y'(0) = 6.$$

3.(15%) Find

$$L^{-1}\{\frac{5}{s} - \frac{3e^{-3s}}{s} - \frac{2e^{-7s}}{s}\},$$

here $L$ means the Laplace transformation.

4.(20%) Solve

$$\frac{dx_1}{dt} = 4x_1 + 3x_2 + x_3,$$

$$\frac{dx_2}{dt} = -4x_1 - 4x_2 - 2x_3,$$

$$\frac{dx_3}{dt} = 8x_1 + 12x_2 + 6x_3.$$

5.(20%)

(1) Find the Wronskian of $t^2y'' - t(t+2)y' + (t+2)y = 0$.

(2) Show that if $p$ is differentiable and $p(t) > 0$, then the Wronskian $W(t)$ of two solutions of $[p(t)y']' + q(t)y = 0$ is $W(t) = c/p(t)$, where $c$ is a constant.

6.(15%) Use the method of reduction of order to solve

$$(2-t)y''' + (2t-3)y'' - ty' + y = 0, \quad t < 2; \quad y_1(t) = e^t.$$

Here $y_1(t)$ is a particular solution.

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