

系所別:

數學系

科目:

微分方程

1.(15%) Solve the equation

$$(x - 2y + 1)dx + (4x - 3y - 6)dy = 0.$$

2.(15%) Given that  $y = x$  is a solution of

$$(x^2 + 1)\frac{dy^2}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$$

Find the general solution of above equation by reducing the order.

3.(20%) Consider the following linear system

$$\frac{dx}{dt} = -2x - y,$$

$$\frac{dy}{dt} = x - 4y.$$

(1) Find the general solution of the linear system.

(2) Find the particular solution of the system with initial condition  $(x_0, y_0) = (1, 0)$ .

4.(10%) Find a particular solution of

$$y''' - 4y' = t + 3 \cos t + e^{-2t}.$$

5.(20%) Consider the following equation with parameter  $a$  by

$$\frac{dy}{dy} = y^2 - ay + 1.$$

Locate the bifurcation values for the one parameter family and draw the phase lines for values of the parameter slightly smaller than, slightly larger than, and at the bifurcation value.

6.(20%) Use Laplace transformation to find the solution of the system

$$\frac{dx}{dt} - 6x + 3y = 8e^t,$$

$$\frac{dy}{dt} - 2x - y = 4e^t,$$

that satisfies the initial conditions

$$x(0) = -1 \quad \text{and} \quad y(0) = 0.$$

參考用