# 國立中央大學113學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生)

統計研究所 碩士班 不分組(在職生)



科目： 數理統計

＊本科考試可使用計算器，廠牌、功能不拘

計算題應詳列計算過程，無計算過程者不予計分

1. Let $f(x,y) = ce^{-y}$, where $c > 0$ and $0 < x < y < \infty$, be the joint probability density function (pdf) of random variables $X$ and $Y$.
   (a) Find the value of $c$. (5%)
   (b) Calculate $P(X+Y>1)$. (5%)
   (c) Compute the conditional variance $\mathrm{Var}(Y|X=x)$, compare it with $\mathrm{Var}(Y)$, and explain your finding. (10%)

2. Let $(X_1, \ldots, X_n)$ be a random sample from a distribution having pdf
$$f(x) = \frac{1}{\theta}e^{-(x-\theta)/\theta},\ x > \theta,$$
   where $\theta > 0$ is an unknown parameter.
   (a) Find a statistic that is minimal sufficient for $\theta$. (10%)
   (b) Show whether the minimal sufficient statistic in (a) is complete. (10%)

3. Let $(X_1, \ldots, X_n)$ be a random sample from $N(\mu, \sigma^2)$ with an unknown $\mu \in \mathcal{R}$ and a known $\sigma^2 > 0$.
   (a) Find the uniformly minimum-variance unbiased estimator (UMVUE) of $e^{t\mu}$ with a fixed $t \neq 0$. (10%)
   (b) Show that the variance of the UMVUE is larger than the Cramér-Rao lower bound. (10%)

4. Let $(X_1, \ldots, X_n)$ be a random sample from a distribution having pdf $f_{\theta,j}$, where $\theta > 0$, $j = 1, 2$, $f_{\theta,1}(x) = (\sqrt{2\pi}\theta)^{-1}\exp\{-x^2/(2\theta^2)\}$, and $f_{\theta,2}(x) = (2\theta)^{-1}\exp\{-|x|/\theta\}$, $x \in \mathcal{R}$.
   (a) Find a maximum likelihood estimator (MLE) of $(\theta, j)$. (10%)
   (b) Show whether the MLE of $j$ in (a) is consistent. (10%)

注意：背面有試題

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5. Let $(X_1, \ldots, X_n)$ be a random sample from $N(\theta, \sigma^2)$, where $\theta \in \mathcal{R}$ and $\sigma^2 > 0$. We are interested in testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ with an unknown $\sigma^2$.

   (a) Show that the test rejecting $H_0$ when

   $$|\overline{X} - \theta_0| > t_{n-1,\alpha/2}\sqrt{S^2/n}$$

   is a likelihood ratio test (LRT) of size $\alpha$, where $\overline{X}$ and $S^2$ are the sample mean and sample variance of the random sample, respectively, and $t_{n,p}$ denotes the $(1-p)$th quantile of a $t$ distribution with $n$ degrees of freedom. (15%)

   (b) Find a $1 - \alpha$ confidence interval for $\theta$ based on (a). (5%)

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