國立中央大學105學年度碩士班考試入學試題

所別： 光電科學與工程學系 碩士班 不分組(一般生)　　　　共1頁　第1頁

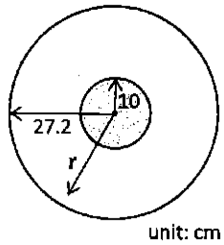
科目： 電磁學

本科考試可使用計算器，廠牌、功能不拘　　　　＊請在答案卷(卡)內作答

1. (10%) For a rectangular coordinate system (unit: m) in free space and use the permittivity of 8.85×10^{-14} F/cm, calculate the electric force acting on a 0.5-mC point charge located at (1, 4, 2), from which there is another point charge of 5 μC at (3, 5, 4).

2. (15%) For a finitely conducting cylinder of radius 2 cm and with a relative permeability of 5, it is found that the magnetic flux density flowing within the cylinder is varying as $1/r\,\bar{a}_\phi$ T, where r is the distance measured from the center of the cylinder, i.e. $0 < r < 2$ cm. If the cylinder is surrounded by free space (permeability: $4\pi\times10^{-9}$ H/cm), determine the magnetic flux just outside the cylinder.

3. (25%) Shown below is the top view of a coaxial cable, and the radii of the inner and the outer conductor are respectively 10.0 cm and 27.2 cm, and $10.0 \text{ cm} \le r \le 27.2$ cm. If the two conductors are surrounded by free space (permittivity: 8.85×10^{-14} F/cm), and the inner conductor is held at a potential of 9 V while the outer conductor is grounded, calculate the following:
   (a) The potential distribution as a function of r between the conductors. (8 %)
   (b) The electric field and the electric flux density distribution between the conductors. (5 %)
   (c) The surface charge density on the inner conductor. (4 %)
   (d) The charge per unit length on the inner conductor. (3 %)
   (e) The capacitance per unit length of the coaxial cable. (5 %)



4. (20%) Regarding the electromagnetic waves in conductors, if the following wave equations and plane-wave solutions are given as follows,

$$\nabla\cdot\bar{E}=0;\quad \nabla\times\bar{E}=-\frac{\partial\bar{B}}{\partial t};\quad \nabla\cdot\bar{B}=0;\quad \nabla\times\bar{B}=\mu\sigma\bar{E}+\mu\varepsilon\frac{\partial\bar{E}}{\partial t}.\qquad \tilde{\bar{E}}=\tilde{\bar{E}}_0e^{i(\tilde{k}z-\omega t)};\quad \tilde{\bar{B}}=\tilde{\bar{B}}_0e^{i(\tilde{k}z-\omega t)}$$

where $\tilde{k}$, σ, μ, and ε are complex wave number ($k+i\kappa$), conductivity, permeability and permittivity, respectively.
   (a) Derive the skin depth (d) in terms of ω, ε, μ, and σ; also describe its physical meaning. (10%)
   (b) Derive the real amplitude ratio of B_0/E_0 in terms of ω, ε, μ, and σ, and describe the physical meaning of a complex wave number. (10%)

5. (20%) Suppose we have a wave guide (filled with air) of rectangular shape with width a and height b, and we are interested in the propagation of TM waves. (a) Find the longitudinal electric field, $E_z(x,y)$, also explain the limitation of m and n in TM_{mn}. (7%) (b) Find the cut-off frequency, and the lowest TM cut-off frequency. (7%) (c) If the wave guide is filled with oil having refractive index of 1.44, is the cut-off frequency the same as that filled with air? Please give your answer in detail. (6%) The following formulas may be helpful.

$$E_x=\frac{i}{(\omega/c)^2-k^2}\left(k\frac{\partial E_z}{\partial x}+\omega\frac{\partial B_z}{\partial y}\right),\quad E_y=\frac{i}{(\omega/c)^2-k^2}\left(k\frac{\partial E_z}{\partial y}-\omega\frac{\partial B_z}{\partial x}\right),\quad \left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{\omega}{c}\right)^2-k^2\right]\begin{matrix}E_z\\B_z\end{matrix}=0$$

$$B_x=\frac{i}{(\omega/c)^2-k^2}\left(k\frac{\partial B_z}{\partial x}-\frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right),\quad B_y=\frac{i}{(\omega/c)^2-k^2}\left(k\frac{\partial B_z}{\partial y}+\frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right),\quad \textit{uncoupled equation}$$

6. (10%) Consider two point charges, $+Q$ and $-Q$, separated by a distance a. Construct the plane equidistant from the two charges. By integrating Maxwell's stress tensor over this plane, determine the force of one charge on the other. $T_{ij}\equiv\varepsilon_0\left(E_iE_j-\frac{1}{2}\delta_{ij}E^2\right)+\frac{1}{\mu_0}\left(B_iB_j-\frac{1}{2}\delta_{ij}B^2\right)$