國立中央大學107學年度碩士班考試入學試題

所別：光電科學與工程學系 碩士班 不分組(一般生)　　共3頁 第1頁

科目：工程數學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答

1) Given that $f(x)=e^{i\left(\sqrt{2}+2i\right)x}$

a) (5pt) Find the argument of this complex function at $x=\frac{\pi}{2\sqrt{2}}$

b) (5pt) Find the argument of $\frac{df(x)}{dx}$ at $x=\frac{\pi}{2\sqrt{2}}$

c) (5pt) Show that the difference of the argument of $f(x)$ and that of $\frac{df(x)}{dx}$ is the same at all points of x.

2) a) (6pt) Find the eigenvectors of the following matrix

$$\begin{pmatrix} 3.75 & -0.433 \\ -0.433 & 3.25 \end{pmatrix}$$

b) (6pt) Draw a figure showing the eigenvectors you have found. Do they have any particular geometric relation between them?

c) (5pt) Calculate the "Dot Product" of the 2 eigenvectors you have found in part-b.

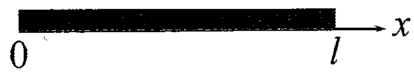
注意:背面有試題

參考用

3) Given that the one-dimensional Heat flow equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad (1)$$

where u stands for temperature, t and x as usual denote respectively the time and the position along the bar in the accompanied diagram; c is a constant. When the boundary and initial conditions for u are



$$u(0,t) = u(l,t) = 0$$
$$u(x,0) = f(x) \qquad (2)$$

it is known that

$$u(x,t) = B_n \sin\frac{n\pi x}{l} e^{-\lambda_n^2 t} \qquad (3)$$

where $\lambda_n = cn\pi / l$ is a particular solution for Eq.(1). Now suppose instead of the above boundary and initial conditions, we have

$$u(0,t) = u(l,t) = U_0 \ \left(U_0 \neq 0 \text{ is a constant} \right)$$
$$u(x,0) = f(x) + U_0 \qquad (4)$$

Show that

a) (9pt) $\left(B_n \sin\frac{n\pi x}{l} e^{-\lambda_n^2 t} + \text{a particular constant} \right)$ is a particular solution of Eq.(1) satisfying the boundary and initial conditions (4).

b) (9pt) What is the particular constant mentioned in part-a? (Show the detail, guessed answer is not accepted)

參考用

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所別： 光電科學與工程學系 碩士班 不分組(一般生)　　共3頁　第3頁

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4) (10pt) Calculate $\det(A)$ and find A^{101} of the following matrix

$$A=\begin{pmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1+i}{\sqrt{2}} \end{pmatrix}$$

5) (10pt) A plane curve is described by the equation

$$29x^2+24xy+36y^2=180$$

What is the shape of this curve? Draw a figure to show this curve.

6) Calculate the divergence of the following two vector fields:

(a) (5pt) $\mathbf{W}=xy\,\hat{i}+yz\,\hat{j}+zx\,\hat{k}$.

(c) (5pt) $\mathbf{Q}=\cos\left(x^2+y^2\right)\nabla e^{x+y+z}$.

7) (10pt) Given that

$$h(x)=u\left(x+\frac{\pi}{2}\right)-u\left(x-\frac{\pi}{2}\right),$$

where $u(x-a)=\begin{cases}1 & \text{if} \quad x\ge a\\ 0 & \text{if} \quad x<a\end{cases}$ is the Heaviside step function.

Find the Fourier transform of the function $f(x)=h(x)\cos 2x$.

8) Solve the following initial value problems. Here $\delta(t-1)$ is the Dirac delta function and $u(t-3)$ is the Heaviside step function.

(a) (5pt) $y''-4y'+3y=\delta(t-1)$, $y(0)=1$, $y'(0)=0$.

(b) (5pt) $y''+y'-2y=u(t-3)$, $y(0)=0$, $y'(0)=1$.

參考用