

國立中央大學九十學年度碩士班研究生入學試題卷

所別: 光電科學研究所 不分組 科目: 應用數學 共 2 頁 第 1 頁

(1) Consider the following mathematical expression

$$\sqrt{x^3 y} + b x^2 \sin \frac{y}{z} \pi = c \quad (c \text{ is a constant}) \quad (1)$$

It describes a surface in the 3-dimensional space. Why is that

- 3% a) the gradient of the function is a vector?
2% b) the gradient is normal to the surface?

Consider $f(x, y, z) = \sqrt{x^3 y} + b x^2 \sin \frac{y}{z} \pi$ (2)

- 2% c) what is the directional derivative $\frac{df}{ds}$ of this function at $x=1, y=2, z=3$ and in the direction $\hat{i} + 2\hat{j} + 3\hat{k}$
2% d) what is the differential increment df of this function at $x=1, y=2, z=3$, for $\Delta x = 10^{-3}$, $\Delta y = 2 \times 10^{-3}$ and $\Delta z = 3 \times 10^{-3}$?
3% e) Can you calculate the same differential increment df in (d) for the surface described in Eq. 1?

(2) There are 2 physical vector quantities U and V that are related to each other by

$$V = \begin{pmatrix} 1 & -6 & 2 \\ -2 & 0 & -1 \\ 2 & -3 & -2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

where $U = \hat{i} u_1 + \hat{j} u_2 + \hat{k} u_3$, $V = \hat{i} v_1 + \hat{j} v_2 + \hat{k} v_3$.

So in general V is not in the same direction as U.

- 9% a) Are there any particular directions along which U and V are parallel to each other. If the answer is "no", why? If "yes", find them.
3% b) Given a particular $U = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$, is this U an eigenvector of the matrix in this problem.

(3) Prove that B_n and c are constant

4% a) $u(x, t) = \sum_{n=1}^{\infty} B_n \cos \frac{cn\pi}{l} t \sin \frac{n\pi x}{l}$ (3)

is the formal solution of the following partial differential equation:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad (4)$$

satisfying the following boundary and initial conditions:

$$u(0, t) = 0 \quad u(l, t) = 0 \quad \text{for all } t$$

$$u(x, 0) = \frac{2Q}{l} x \quad \text{when } 0 < x < \frac{l}{2}$$

$$= \frac{2Q}{l} (l-x) \quad \text{when } \frac{l}{2} < x < l$$

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and $\frac{\partial u}{\partial t} \Big|_{t=0} = 0$

9% b) Can you find the B_n ? Find the B_1 , B_2 and B_3 explicitly.

4% c) Is the $u(x,t)$ in expression 3 with the B_n 's given in b) a solution of 4 at every point in the range? Yes or No? If "yes", why? If "no", why?

(4) Given $S_1 = (1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}) + (\frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8}) + \dots$

3% a) Is S_1 a convergent series? Give reason.

S_2 is a series obtained by regrouping series S_1

$$S_2 = (1 + \frac{1}{3} - \frac{1}{2}) + (\frac{1}{5} + \frac{1}{7} - \frac{1}{4}) + \dots$$

$$= \sum_{m=1}^{\infty} \left(\frac{1}{4m-3} + \frac{1}{4m-1} - \frac{1}{2m} \right)$$

3% b) Is S_2 a convergent series, give reason.

3% c) Given $S_1' = (1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}) + (\frac{1}{5^2} - \frac{1}{6^2} + \frac{1}{7^2} - \frac{1}{8^2}) + \dots$

and $S_2' = (1 + \frac{1}{3^2} - \frac{1}{2^2}) + (\frac{1}{5^2} + \frac{1}{7^2} - \frac{1}{4^2}) + \dots$

$$= \sum_{m=1}^{\infty} \left(\frac{1}{(4m-3)^2} + \frac{1}{(4m-1)^2} - \frac{1}{(2m)^2} \right)$$

It is known already that $S_1 \neq S_2$. what about S_1' and S_2' ; are they equal? (只有答案, 没有理由, 错了, 令倒扣分数)

(5) (15%) 解初值問題

$$4y'' + 4y' + y = 0, \quad y(0) = -2, \quad y'(0) = 1$$

(6) (15%) 求解

$$y'' + 6y' + 9y = e^{-3x}/(x^2+1)$$

(7) (20%) 用 Laplace transform 解

$$y'' - 3y' + 2y = 6e^{-x}, \quad y(0) = 3, \quad y'(0) = 3$$

