所別：光電科學研究所碩士班 一般生 科目：電磁學
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共七題，第一二題為選擇題，請把你的答案寫在答案卷上.

For problem 1 and 2:

A thin sheet of perfect metal is located at $z = 0$ in vacuum and set as zero electric potential. The magnetic field at point $A(0, 0, h)$ is measured as H_0 along $+y$-direction and so does every point with $z = h$. Please solve problem 1 and 2 with this geometry.

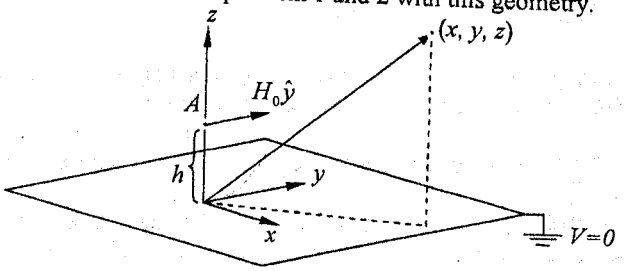

Figure 1. Perfect metal sheet in space at $z = 0$ with potential 0 for problem 1 – 4.

1. (15 pts total) If a point charge Q is placed at point A:

2% (1a) (2 pts) What's the surface current density distribution on this metal sheet?

(a) $\frac{\mu_0 H_0}{\sqrt{2h^2}}\hat{x}$, (b) $\frac{-H_0}{h^2}\hat{y}$, (c) $\frac{-H_0}{h}\hat{x}$, (d) $-4\pi\mu_0\sqrt{h}H_0\hat{x}$, (e) $\frac{\sqrt{h}H_0}{2\pi}\hat{y}$ (f) none of above

2% (1b) (2 pts) What's the magnetic field at an arbitrary point (x, y, z) in space?

(a) $\frac{z^2 H_0}{h^2}\hat{y}$, (b) $\frac{zH_0}{h}\hat{y}$, (c) $\frac{h^2 H_0}{z^2}\hat{y}$, (d) $\frac{h^2 H_0}{x^2+y^2+z^2}\hat{y}$, (e) $\frac{hH_0}{z}\hat{y}$, (f) $\frac{hH_0}{\sqrt{x^2+y^2+z^2}}\hat{y}$

4% (1c) (4 pts) What's the value of electric field at an arbitrary point (x, y, z) in space for $z < 0$?

(a) $\frac{1}{4\pi\varepsilon_0}\frac{hQ}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$, (b) $\frac{1}{4\pi\varepsilon_0}\frac{hQ}{x^2+y^2+(z-h)^2}$, (c) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x^2+y^2+(z-h)^2}-\frac{1}{x^2+y^2+(z+h)^2}\right]$, (d) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2+y^2+(z-h)^2}$, (e) $\frac{1}{4\pi\varepsilon_0}\frac{-Q}{x^2+y^2+(z-h)^2}$ (f) none of above

3% (1d) (3 pts) What's the surface charge density distribution on the metal sheet?

(a) $\frac{1}{2\pi}\frac{-Qh}{\left(x^2+y^2+h^2\right)^{\frac{3}{2}}}$, (b) $\frac{1}{2\pi h}\frac{Qe^{-\frac{h^2}{x^2+y^2}}}{x^2+y^2}$, (c) $\frac{1}{2\pi}\frac{-Q}{x^2+y^2}$, (d) $\frac{1}{2\pi h}\frac{Q}{\sqrt{x^2+y^2}}$, (e) $\frac{1}{2\pi}\frac{-2Q}{\left(x^2+y^2+h^2\right)^{\frac{3}{2}}}$, (f) none of above

4% (1e) (4 pts) If this charge can move at a constant velocity without external force applied on it, please solve this velocity. (No gravitational force is applied.)

(a) $\frac{1}{4\pi\varepsilon_0}\frac{-Q}{4h}\hat{y}$, (b) $\frac{1}{4\pi\varepsilon_0\mu_0}\frac{-Q^2}{h^2}\hat{x}$, (c) $\frac{1}{4\pi\varepsilon_0}\frac{-Q^2}{h}\hat{z}$, (d) $\frac{1}{4\pi\varepsilon_0}\frac{-Q^2}{2h}\hat{y}$, (e) $\frac{1}{4\pi\varepsilon_0\mu_0}\frac{Q}{4h^2}\hat{x}$, (f) none of above

2. (20 pts. total) If the line charge is replaced by a loop with radius a with current I on it:

2% (2a) (2 pts.) When the loop's magnetic dipole moment is pointing along $+z$-direction, what's the direction of the net force applied on it? (a) $+\hat{x}$, (b) $+\hat{y}$, (c) $+\hat{z}$, (d) $-\hat{x}$, (e) $-\hat{y}$, (f) $-\hat{z}$, (g) zero net force.

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2% (2.b) (2 pts.) and what's the net toque direction? (a) $+\hat{x}$, (b) $+\hat{y}$, (c) $+\hat{z}$, (d) $-\hat{x}$, (e) $-\hat{y}$, (f) $-\hat{z}$, (g) zero net toque.

2% (2.c) (2 pts.) When the loop's magnetic dipole moment is pointing along $+y$-direction, what's the direction of the total force applied on it? (a) $+\hat{x}$, (b) $+\hat{y}$, (c) $+\hat{z}$, (d) $-\hat{x}$, (e) $-\hat{y}$, (f) $-\hat{z}$, (g) zero net force.

2% (2.d) (2 pts) and what's the net toque direction? (a) $+\hat{x}$, (b) $+\hat{y}$, (c) $+\hat{z}$, (d) $-\hat{x}$, (e) $-\hat{y}$, (f) $-\hat{z}$, (g) zero net toque

3% (2.e) (3 pts) For the loop having magnetic dipole direction along $-z$-direction, calculate the direction and value of the vector potential at (0, 0, 0).

3% (2.f) (3 pts) and the direction and value of the magnetic flux density at (0, 0, 0).

3% (2.g) (3 pts) For the loop having magnetic dipole direction along $-y$-direction, calculate the direction and value of the vector potential at (0, 0, 0).

3% (2.h) (3 pts) and the direction and value of the magnetic flux density at (0, 0, 0).

15% 3. (15 pts.) Calculate the mutual inductance per unit length between two parallel two-wire transmission lines as shown in Fig. 2.

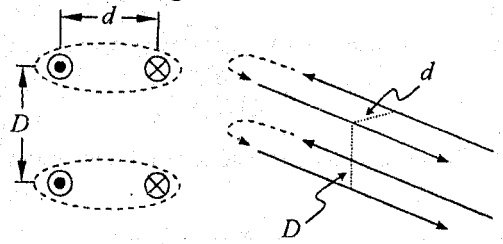

Figure 2. Two parallel two-wire transmission lines configuration for problem 3.

(4)

5% a) 寫出 the mathematical expression of the **instantaneous** Poynting vector and explain what it is . What is its unit? Why is it in general we don't use the instantaneous value of it but use its **average** value? (5%)

5% b) Explain what are TE wave and TM wave, respectively, in a wave guide. (5%)

4% c) A beam of He-Ne laser light(λ =632.8 nm in air)enters a hollow(中空)wave guide made of infinitely conducting material. The laser beam is found to keep on propagating along the guide with its plane of constant phase perpendicular to the guide axis. Again the medium in the guide is air. 請問此時之波長是否仍然是 632.8 nm?(此題只需回答〝是〞抑或〝不是〞，不需說明，但答錯會倒扣 2 分)

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16% (5) Find (A) the intensity or irradiance(光強) in units of watt/m^2 and (B) the electric field measured perpendicular to the propagation direction of the light waves generated by a 60 watt light bulb(燈泡) at 3 meters away from the light bulb, assuming(see Fig.3)

a) the light bulb is a point source,

b) only 20% of the electric power it consumes is converted to light power,

c) the light waves are plane waves at that distance.

Given $\varepsilon_o = 8.85\times10^{-12}$ farad/m and $\mu_o = 4\pi\times10^{-7}$ henry/m

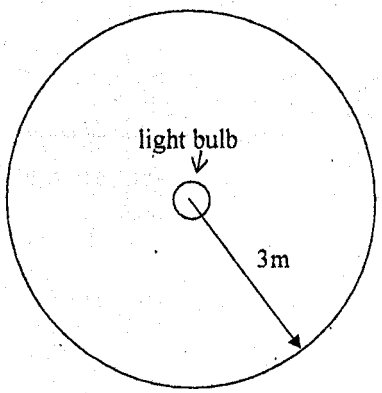

Fig.3

12% (6) It can be shown that the equation governing the dynamic E-field and B-field in a conductor having permittivity, permeability and conductivity ε, μ and σ, respectively, are

$$\nabla^2 E - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} - \mu\sigma\frac{\partial E}{\partial t} = 0$$

$$\nabla^2 B - \mu\varepsilon\frac{\partial^2 B}{\partial t^2} - \mu\sigma\frac{\partial B}{\partial t} = 0$$

(a) 請導出通過這種介質的平面波的 propagation vector 和頻率之間的關係式。

(b) Is the plane wave mentioned in part (a) a transverse wave or longitudinal wave? Prove it. 你只需證明其中之 E-field 或 B-field 即可。

(7) Consider a plane wave incident on an interface of two media as shown in Fig. 4, Fig.5 shows the relationship between $\vec{E}_{ot}$, $\vec{E}_{or}$ and $\vec{E}_{oi}$ when they are lying on the

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incident plane, where $\vec{E}_{ot}, \vec{E}_{or}$ and $\vec{E}_{oi}$ are the amplitudes of the transmitted , reflected and incident waves, respectively. Suppose Fig. 6 shows the E-field of the incident wave with respect to time at point A in Fig. 4 (a) Copy Fig. 6 in your answer sheet(答案卷) and draw the corresponding reflected waves below it. (5%)

(b) There is a technical term (科學名稱) for the angle in point B in Fig. 5. What is it? (答錯倒扣 1.5 分) (3%)

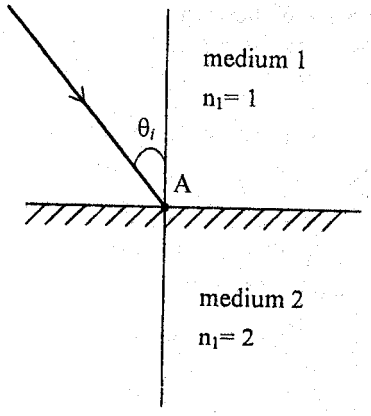

Fig. 4

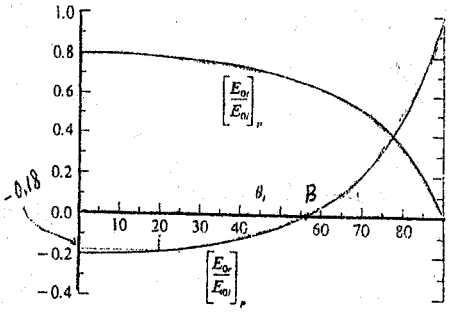

Fig. 5

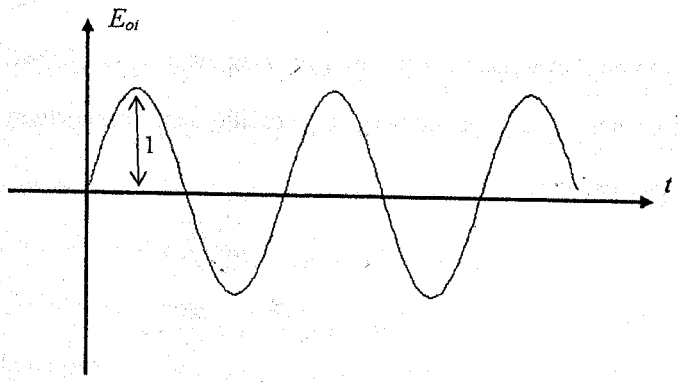

Fig. 6