國立中央大學98學年度碩士班考試入學試題卷

所別：光電科學與工程學系碩士班 一般生　科目：近代物理　共 3 頁 第 1 頁

＊請在試卷答案卷（卡）內作答

Please use the following notations (標記、符號) for your answers.

h: Planck's constant

e: Electron charge

m: Mass of a particle

c: Speed of light

ν: Frequency

λ: Wavelength

1. A particle has mass m at rest in the beginning. After a certain period of time, it decays into two identical particles. Each particle has rest mass of $A \cdot m$ where $A<0.5$. Consider this as a relativistic system. (Fill-in-the-blank problems, please just give the result using the notations and A.)

(a) (5%) The momentum of each particle at the final state is _____.

(b) (5%) The speed of each particle at the final state is _____.

2. A particle with mass m is placed in a **rigid box**. **Use uncertainty principle** to calculate the lowest possible energy. (Fill-in-the-blank problems, please just give the result.)

(a) (5%) If the rigid box is **1D** and the length is a, the lowest possible energy is _____.

(b) (5%) If the rigid box is **3D** and cubical with the each side length of a, the lowest possible energy is _____.

3. (10%) ***Sketch*** all the energy levels of **Helium ion** (He^+) from n=1 to n=3 in the order or low energy to high energy and draw all the allowed transitions according to selection rules.

4. (5%) Neils Bohr explained the hydrogen atom spectrum and started the era of quantum mechanics. Can you describe the properties of the spectrum given by **hydrogen *ion***? Explain your answer with the knowledge you've learnt from modern physics (**limited to 100 words or less**).

5. 1-D finite asymmetry potential well is shown in Fig. A labeled as $U(x)$ on the upper left corner.

(a) (5%) Choose the possible wavefunctions/probabilities that could be the **steady state** wavefunctions (**Any wrong guessing will have penalty of -1 pt**).

_____

參考用

注意：背面有試題

(b) (5%) What are the rules that you had used to consider your answers in (a) are correct? Please write down the rules based on modern physics you've learnt.

(c) (5%) Please ***sketch*** the wavefunction of the particle if the total energy of the particle is in between β and γ assuming the particle **can only have 4 steady states** in the well.

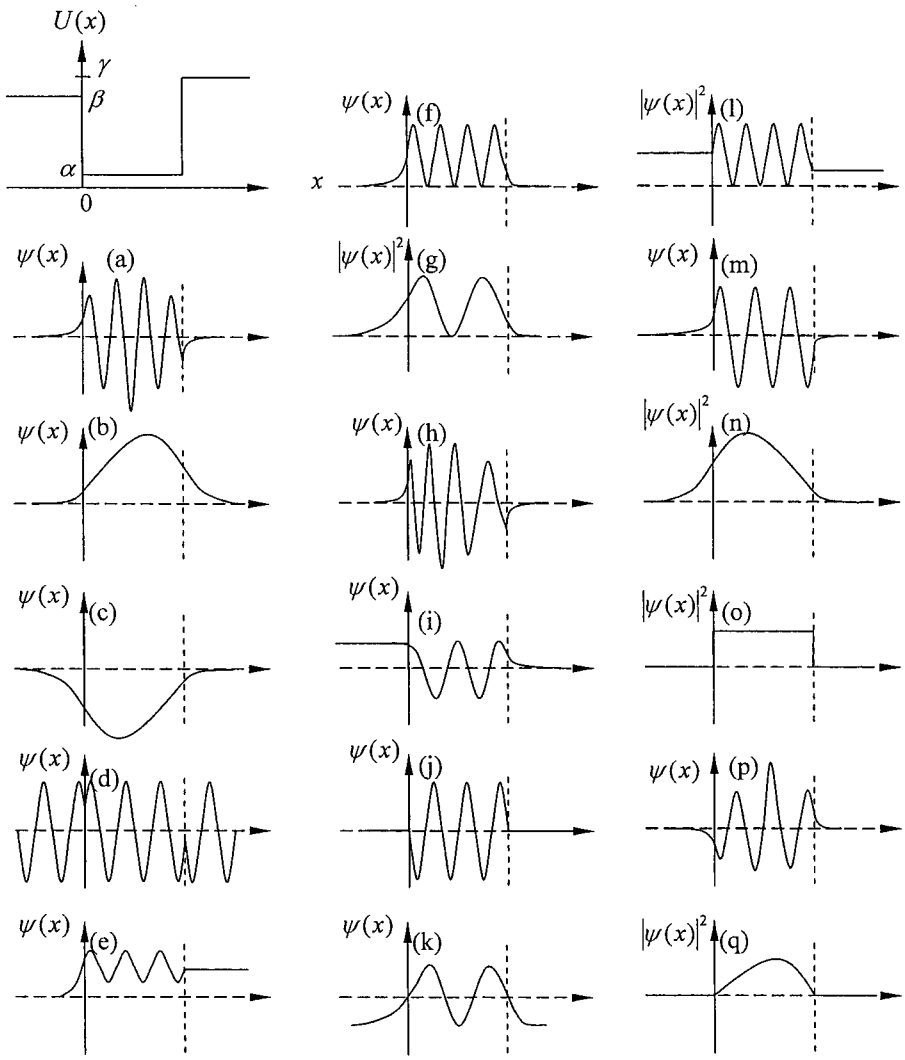

Fig. A. The asymmetry potential well and the possible wavefunction/probability distributions.

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6. Figure B shows a typical X-ray spectrum emitted from an X-ray tube using a Mo anode.

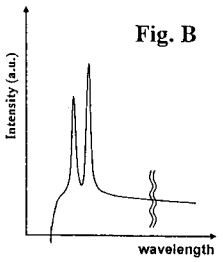

Fig. B

(a) (5%) Please explain the physics and mechanisms dominant in producing such a spectral curve.

(b) (6%) Design and sketch an experimental setup for acquiring the spectrum of an X-ray source like the one obtained in Fig. B. Illustrate the methodology used in your design.

(c) (5%) Propose a scheme to use your setup in (b) to further demonstrate the Compton effect.

(d) (5%) Qualitatively predict (please explain) and plot the Compton scattering spectrum obtained from your scheme given in (c).

7. (10%) A photon of frequency υ was emitted from a star of mass M_A and radius R_A to a star of mass M_B and radius R_B. The center-to-center distance between the two stars is R. Assuming the two stars are far away from other stars in the universe. Please find the frequency of the photon when it travels to the middle of the two stars.

8. (8%) The emission spectrum of a wave (laser) has a Gaussian amplitude distribution over a bandwidth (full width at half maximum (FWHM)) of $\Delta\nu$ containing a set of discrete frequencies with a frequency spacing of $\Delta\nu_m$, as shown in Fig. C. If all the frequency components are in phase, please estimate the smallest possible FWHM width of a temporal pulse that can be generated with this wave. (Hint: the field amplitude of the frequency component ν in the Gaussian profile can be expressed as $E^2(\nu) = E_0^2 \exp[-\left(\frac{2|\nu-\nu_0|}{\Delta\nu}\right)^2 \ln 2]$, where E_0 is the field amplitude of the central frequency ν_0.)

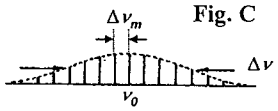

Fig. C

9. (5%) Deuterium (^{2}H) is an isotope of the hydrogen (^{1}H). Though they have the same nuclear charge, they exhibit some slight difference in the emission spectra due to the so called "reduced mass" effect. Please estimate the wavelength difference ratio of a specific spectral line emitted from them (*i.e.* estimate $(\lambda(^1H) - \lambda(^2H))/\lambda(^1H)$) = _____. (Fill-in-the-blank problem, please just give the result.)

10. (6%) Please write a short essay interpreting the contribution of Max Planck's hypothesis made in 1901 to the blackbody radiation _____ (Fill-in-the-blank problem, please just give the result) (**limited to 80 words or less**).

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