

國立中央大學 105 學年度碩士班考試入學試題

所別： 天文研究所碩士班 不分組(一般生)

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天文研究所碩士班 不分組(在職生)

科目： 普通物理

本科考試禁用計算器

*請在答案卷(卡)內作答

1. (Total 20%) There are four particles whose positions can be written as functions of time t as

$$\bar{x}_1(t) = (A + Bt)\hat{x} + (C + Dt)\hat{y} + (E + Ft)\hat{z}$$

$$\bar{x}_2(t) = (V_x t)\hat{x} + \left(V_y t - \frac{1}{2}gt^2 \right)\hat{y} + (V_z t)\hat{z}$$

$$\bar{x}_3(t) = [R \cos(\omega t)]\hat{x} + [R \sin(\omega t)]\hat{y} + (v_z t)\hat{z}$$

$$\bar{x}_4(t) = [x_0(1 - e^{-\alpha t})]\hat{x} + y_0\hat{y} + z_0\hat{z}$$

where $A, B, C, D, E, F, V_x, V_y, V_z, g, R, \omega, v_z, \alpha, x_0, y_0$ and z_0 are constants

- (i) (10%) Find the velocities and accelerations of these four particles.
(ii) (10%) Suppose all four particles have identical mass m , find the powers of the external forces acting on these four particles individually.
2. (Total 20%) A planet moves around the Sun in an elliptical orbit. Suppose the mass of the planet m_p is much smaller than the mass of the Sun M_\odot , that is $m_p \ll M_\odot$, so the motion of the Sun can be neglected. The orbit of the planet can be written in the polar coordinate as

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$

where a is the semimajor axis of the ellipse, e is the eccentricity and the Sun is located at the origin where is also one of the foci of the ellipse. Both energy and angular momentum are conserved in this system.

- (i) (5%) We call the planet is at the perihelion (近日點) where the planet is closest to the Sun, $r = r_p$, and at the aphelion (遠日點) where the planet is farthest to the Sun, $r = r_a$. Find the r_p and r_a as functions of a and

e .

注意：背面有試題

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(ii) (5%) Using the conservation of angular momentum and result from (i), find v_p/v_a where v_p and v_a are the speeds of the planet at perihelion and aphelion, respectively.

(iii) (5%) Using the conservation of energy and the results from (i) and (ii), find the v_p and v_a .

(iv) (5%) Prove that the total energy of the system is

$$E_{tot} = -\frac{GM_{\odot}m_p}{2a}$$

3. (Total 15%) A vacuum box of temperature T contains a blackbody radiation field in it. The internal energy per unit volume is $u = U/V = aT^4$ where V is the volume of the box and a is the radiation constant. The radiation pressure is

$$P = \frac{u}{3} = \frac{1}{3}aT^4.$$

(i) (10%) Using the first-law of thermodynamics, find the entropy S as a function of temperature T and volume of the box V if we define $S = 0$ for $T = 0$.

(ii) (5%) Prove the law of adiabatic expansion for this system is

$$PV^{4/3} = const$$

4. (Total 20%) There are two concentric spheres of radii R_1 and R_2 where

$R_1 < R_2$ with charges Q_1 and Q_2 , respectively.

(i) (5%) Find the potential and electric field for $r > R_2$, $R_2 > r > R_1$ and

$r < R_1$.

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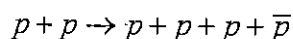
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- (ii) (5%) Find the total potential energy of the system.
- (iii) (5%) Now there is a conducting wire connecting two spheres. After the system reaching the new equilibrium state, what are the charges on both spheres?
- (iv) (5%) How much energy is released after the wire being connected?
5. (10%) The rest mass of a proton is 931 MeV. If we want to make an antiproton in laboratory through the following process



by accelerating a proton (proton A) to collide another proton (proton B) rest in laboratory, what is the minimum kinetic energy required for the proton A?

6. (Total 15%) The Hamiltonian of a simple harmonic oscillation system can be written as

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

and the ground state wave function is

$$\psi(x) = A \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

where $-\infty < x < \infty$ and A is the normalization constant.

- (i) (5%) Find the normalization constant A .
- (ii) (5%) Find the expectation values of $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$ and $\langle p^2 \rangle$.
- (iii) (5%) Find the ground state energy of this system

(Hint: You may use the following mathematic identities for your calculations)

$$\int_{-\infty}^{\infty} \exp(-u^2) du = \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} u^2 \exp(-u^2) du = \frac{\sqrt{\pi}}{2}$$