國立中央大學105學年度碩士班考試入學試題

所別：天文研究所 碩士班 不分組(一般生)　　　　共3頁　第1頁
天文研究所 碩士班 不分組(在職生)

科目：普通物理

本科考試禁用計算器　　　　＊請在答案卷(卡)內作答

1. (Total 20%) There are four particles whose positions can be written as functions of time t as

$$\vec{x}_1(t) = (A+Bt)\hat{x} + (C+Dt)\hat{y} + (E+Ft)\hat{z}$$
$$\vec{x}_2(t) = (V_x t)\hat{x} + \left(V_y t - \frac{1}{2}gt^2\right)\hat{y} + (V_z t)\hat{z}$$
$$\vec{x}_3(t) = [R\cos(\omega t)]\hat{x} + [R\sin(\omega t)]\hat{y} + (\mathrm{v}_z t)\hat{z}$$
$$\vec{x}_4(t) = \left[x_0\left(1-e^{-\alpha t}\right)\right]\hat{x} + y_0\hat{y} + z_0\hat{z}$$

where $A, B, C, D, E, F, V_x, V_y, V_z, g, R, \omega, \mathrm{v}_z, \alpha, x_0, y_0$ and z_0 are constants

(i) (10%) Find the velocities and accelerations of these four particles.

(ii) (10%) Suppose all four particles have identical mass m, find the powers of the external forces acting on these four particles individually.

2. (Total 20%) A planet moves around the Sun in an elliptical orbit. Suppose the mass of the planet m_p is much smaller than the mass of the Sun $M_\odot$, that is $m_p \ll M_\odot$, so the motion of the Sun can be neglected. The orbit of the planet can be written in the polar coordinate as

$$r = \frac{a\left(1-e^2\right)}{1+e\cos\theta}$$

where a is the semimajor axis of the ellipse, e is the eccentricity and the Sun is located at the origin where is also one of the foci of the ellipse. Both energy and angular momentum are conserved in this system.

(i) (5%) We call the planet is at the perihelion (近日點) where the planet is closest to the Sun, $r = r_p$, and at the aphelion (遠日點) where the planet is farthest to the Sun, $r = r_a$. Find the r_p and r_a as functions of a and e.

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(ii) (5%) Using the conservation of angular momentum and result from (i), find v_p/v_a where v_p and v_a are the speeds of the planet at perihelion and aphelion, respectively.

(iii) (5%) Using the conservation of energy and the results from (i) and (ii), find the v_p and v_a.

(iv) (5%) Prove that the total energy of the system is

$$E_{tot} = -\frac{GM_{\odot}m_p}{2a}$$

3. (Total 15%) A vacuum box of temperature T contains a blackbody radiation field in it. The internal energy per unit volume is $u = U/V = aT^4$ where V is the volume of the box and a is the radiation constant. The radiation pressure is $P = \frac{u}{3} = \frac{1}{3}aT^4$.

(i) (10%) Using the first law of thermodynamics, find the entropy S as a function of temperature T and volume of the box V if we define $S = 0$ for $T = 0$.

(ii) (5%) Prove the law of adiabatic expansion for this system is

$$PV^{4/3} = const$$

4. (Total 20%) There are two concentric spheres of radii R_1 and R_2 where $R_1 < R_2$ with charges Q_1 and Q_2, respectively.

(i) (5%) Find the potential and electric field for $r > R_2$, $R_2 > r > R_1$ and $r < R_1$.

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(ii) (5%) Find the total potential energy of the system.

(iii) (5%) Now there is a conducting wire connecting two spheres. After the system reaching the new equilibrium state, what are the charges on both spheres?

(iv) (5%) How much energy is released after the wire being connected?

5. (10%) The rest mass of a proton is 931 MeV. If we want to make an antiproton in laboratory through the following process

$$p+p \rightarrow p+p+p+\overline{p}$$

by accelerating a proton (proton A) to collide another proton (proton B) rest in laboratory, what is the minimum kinetic energy required for the proton A?

6. (Total 15%) The Hamiltonian of a simple harmonic oscillation system can be written as

$$H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2$$

and the ground state wave function is

$$\psi(x)=A\exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

where $-\infty<x<\infty$ and A is the normalization constant.

(i) (5%) Find the normalization constant A.

(ii) (5%) Fine the expectation values of $\langle x\rangle, \langle p\rangle, \langle x^2\rangle$ and $\langle p^2\rangle$.

(iii) (5%) Find the ground state energy of this system

(Hint: You may use the following mathematic identities for your calculations

$$\int_{-\infty}^{\infty}\exp\left(-u^2\right)du=\sqrt{\pi}$$

$$\int_{-\infty}^{\infty}u^2\exp\left(-u^2\right)du=\frac{\sqrt{\pi}}{2})$$