(1) (20 points)

In classical mechanics, the angular momentum of an object is given as $L = I\Omega$, where I is the moment of inertia and ω is the angular velocity.

(a) (10 points) For a spherical body with radius R and mass M, please show that $I = \frac{2}{5}MR^2$, if its density is uniform with $\rho = \rho_0 = $ constant everywhere.

(b) (10 points) For a sphere with a two-layer structure, we have $\rho = 2\rho_0$ for $0 < r \le \frac{1}{2}R$, and $\rho = \rho_0$ for $\frac{1}{2}R < r \le R$. Please compute the moment of inertia of this sphere in terms of its total mass M and R.

(2) (20 points)

According to astronomical observations, the rotation periods of cometary nuclei have an upper limit of $P_c = 6$ hours or 360 minutes.

(a) (10 points) The rotation period of a spherical cometary nuclei is $250\sqrt{2}$ minutes. If the interiors of cometary nuclei are composed of small pieces without material strength, please show that the average density of cometary nuclei should be larger than $\rho = 100\pi$ kg m^{-3}.

(b) (10 points) If the radius of a cometary nucleus of spherical shape is $R = 4$ km, density $\rho = 100\pi$ kg m^{-3}, what is its surface gravity, and what is the surface escape velocity?
[Note: Take gravitational constant $G = \frac{2}{3} \times 10^{-10}$ m^3 kg^{-1} s^{-2}]

(3) (20 points)

Subatomic particle A decays at rest to two particles B and C, A $\to$ B + C. The rest mass of A, B and C are M_A, M_B and M_C.

(a) (5 points) What is the relation between the kinetic energy and momentum of a particle of mass M in relativistic mechanics? What is the relation in non-relativistic limit? (Please define "non-relativistic".)

(b) (10 points) If $M_A = \sqrt{10}\,u$, $M_B = u$, $M_C = 2\,u$, (where u is the atomic mass unit), what is the speed of particle B in unit of the speed of light c? (Hint: This is in the relativistic regime.)

(c) (5 points) Now if particle B is a photon and particles A and C have the same mass as in (b), what is the momentum and energy of particle B?

(4) (10 points)

From a medium with refraction index n_1, a light beam enters another medium with refraction index n_2, where $n_1^2 = n_2 = 2$.

(a) (5 points) If the incident angle $\theta_1 = 45°$, please give the value of the exit angle θ_2.

(b) (5 points) If the light beam goes from the medium with n_2 to the other medium with n_1, what is the critical angle of incidence θ_c if this light beam is to be totally reflected without transmission.

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參考用

(5) (15 points)
   (a) (5 points) Explain the first and the second law of thermodynamics.
   (b) (10 points) The working substance of a Carnot engine is an ideal gas. It operates between two temperature reservoirs, one at higher temperature T_H and the other at lower temperature T_L. The engine cycles between four states, A, B, C and D: (i) expands isothermally from A to B, (ii) expands adiabatically from B to C, (iii) compresses isothermally from C to D, and (iv) compresses from D back to A. Find the work done, entropy change and the efficiency of the engine in one cycle.

(6) (15 points)
   (a) (7 points) Write down Maxwell's equations in vacuum (i.e., no sources of charges and currents). Show that both electric field and magnetic field satisfy the wave equation. What is the phase speed of the wave.
   (b) (8 points) Suppose the electric field is described by $\mathbf{E} = \mathbf{E}_o \cos(\mathbf{k} \cdot \mathbf{x} - \omega t)$. Find the magnetic field and the relation between $\mathbf{k}$ and ω.

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