國立中央大學八十五學年度碩士班研究生入學試題卷

所別: 天文研究所 不分組 科目: 物理學 共 1 頁 第 1 頁

(1) (20 points)
A particle of mass m is subject to a force $-\hat{e}_r \mathrm{d}U(r)/\mathrm{d}r$, where $\hat{e}_r$ is the unit position vector, r is the distance from the origin, and $U(r)$ is an arbitrary function of r.
(i) Show that the total energy E and angular momentum L of the particle are conserved.
(ii) Sow that $m\mathrm{d}^2r/\mathrm{d}t^2 = -\mathrm{d}V(r)/\mathrm{d}r$ and find the function $V(r)$.
Given that $U(r) = -U_o(r_o/r)^n$, where U_o and r_o are constants.
(iii) If the criterion for a circular orbit is $\mathrm{d}V(r)/\mathrm{d}r = 0$, what is the radius of the circle?
(iv) What is the criterion on n for a stable circular orbit?

(2) (20 points)
There are three types of damped oscillator: underdamping, overdamping and critical damping.
(i) Describe these three types of motions.
One of two identical masses is attached to a spring with a spring constant k_1, and the other is attached to another spring with a spring constant k_2. The two oscillators are coupled by friction, which is proportional to the relative velocity between the masses.
(ii) Write down the equations of motion of the system.
(iii) What is the characteristic equation for the eigenfrequencies?
(iv) Describe the coupled oscillations of the system if (a) $k_1 \neq k_2$, and (b) $k_1 = k_2$.

(3) (20 points)
(i) Show that charge conservation is implied by the Maxwell's equations.
(ii) Show that in vaccuum (i.e., no charges and currents) both electric field and magnetic field satisfy the wave equation, and find the phase speed of the wave.
Suppose the electric field of an electromagnetic wave in vacuum is given by $\mathbf{E} = \mathbf{E}_o \cos(\mathbf{k} \cdot \mathbf{x} - \omega t)$.
(iii) Find the magnetic field and the relation between $\mathbf{k}$ and ω.
(iv) (a) What are the magnitude and direction of the momentum density and the energy flux of the electromagnetic wave? (b) What is the average energy flux of the wave?

(4) (20 points)
An electron is moving in a circular orbit around a nucleus with charge Ze.
(i) Assume that the nucleus is stationary, what is the total energy of the electron in terms of the radius of the circle?
The energy change of an electron through radiation is given by $\mathrm{d}E/\mathrm{d}t = -e^2|\mathbf{a}|^2/(6\pi\epsilon_o c^3)$, where $\mathbf{a}$ is the acceleration of the electron. Suppose that the fractional energy change by radiation in one orbital period is very small.
(ii) What is the time needed for the electron to change from a total energy E_o to E_1?
(iii) Describe the motion of the electron above.
(iv) Describe the motion of an electron in a uniform magnetic field (a) with, and (b) without radiation.

(5) (20 points)
(i) Write down the ideal gas law and the first law of thermodynamics.
(ii) Taking the entropy and internal energy as functions of temperature and volume (i.e., $S = S(T, V)$ and $U = U(T, V)$), show that the internal energy of an ideal gas depends on temperature only.
The number density and pressure of a gas are defined as: $n = \int f(p)\,\mathrm{d}^3p$ and $P = \frac{1}{3}\int vpf(p)\,\mathrm{d}^3p$, where v and p are the velocity and momentum, and the integration is taken over the whole momentum space. Suppose that the distribution function satisfies Maxwell-Boltzmann statistics.
(iii) Show that the gas satisfies the ideal gas law.
(iv) What is the average of $p^2/2m$?