

國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 天文研究所 不分組 科目: 應用數學 共 / 頁 第 / 頁

(1) (a) Evaluate the integral: $\int_0^{\infty} e^{-x} \cos(2x) dx$. (10%)

(b) Evaluate the integral: $\int_0^{\infty} e^{-x} \sin(2x) dx$. (10%)

(c) Evaluate the integral to the second order in ϵ : $\int_0^1 [e^{\epsilon x} / \cos(\epsilon x)] dx$, where $\epsilon \ll 1$. (10%)

(2) (a) Solve for the differential equation (15%):

$$\frac{df}{dx} - \frac{2f}{x} = 0.$$

(b) Solve for the differential equation (15%):

$$\frac{df}{dx} - \frac{2f}{x} = x^2.$$

(3) Show that $\tan(ix) = i \tanh(x)$, where $i \equiv \sqrt{-1}$. (20%).

(4) Show that $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$, where \cosh^{-1} is the inverse function of \cosh . (20%)