

國立中央大學九十學年度碩士班研究生入學試題卷

所別: 天文研究所 不分組 科目: 應用數學 共 1 頁 第 1 頁

1. (a.) (15%) Find the general solution of the differential equation

$$y'' - 4y' + 3y = 0, \quad \text{and}$$

- (b) (10%) solve the equation for the initial values

$$y(0) = 1, \quad y'(0) = 5.$$

2. (a.) (15%) Use the method of matrix, eigen values and eigen vectors to find the general solution of the system of differential equations

$$x'' = -37x + 12y$$

$$y'' = 12x - 37y, \quad \text{and}$$

- (b.) (10%) find the solution for the initial condition that

$$x(0) = 1, \quad y(0) = 0, \quad x'(0) = 0, \quad y'(0) = 0.$$

3. (25%) Find the Fourier series of the functions $f(x)$ and $g(x)$ which are assumed to have the period 2π , where

$$f(x) = 2x \quad (-\pi < x < \pi)$$

$$g(x) = x^2 \quad (-\pi < x < \pi).$$

Is there any relation between the coefficients of the above two series?

- 4.(a). (10%) Find the value k of the probability distribution function, such that its integration over the whole real axis is 1.

$$f(x) = k / (4x^2 + 25).$$

- (b.) (15%) Evaluate the following integral by the residue theorem.

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^4}$$