科目：近代物理　　校系所組：中大物理學系、天文研究所　清大物理學系、先進光源學程甲組　交大電子物理學系丙組、物理研究所、分子科學研究所

1. (1) Given a potential $V(\mathbf{r}, t)$, first write down the Hamiltonian H and then the Schrödinger equation for the three dimensional space. (10%)

   (2) For a potential $V = V(\mathbf{r})$ that does not depend on t, show that the solution of the Schrödinger equation can be expressed as $\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) f(t)$. What is the form of $f(t)$? (10%)

   (3) Is it always true that $H\,\Psi(\mathbf{r}, t) = E\,\Psi(\mathbf{r}, t)$? Explain your answer. ($E$ is the energy of the system.) (10%)

2. The wavefunction of a particle with mass m in a one-dimensional infinite square well of width a, $x \in (0, a)$, and at time $t = 0$, is given by $\Psi(x, 0) = \sqrt{\frac{2}{7}}\,\psi_1(x) + \sqrt{\frac{5}{7}}\,\psi_2(x)$, where $\psi_1(x)$ and $\psi_2(x)$ are the ground state and first excited stationary state of the system.

   (1) Write down the wavefunction $\Psi(x, t)$ at time t **explicitly** in terms of a and m. (10%)

   (2) You measure the energy of an electron at time $t = 0$. Write down the possible values of the energy and the probability of measuring each. (5%)

   (3) Find the expectation value of the energy in the state $\Psi(x, t)$ above at any given t. (5%)

3. In a light atom, two electrons are to occupy the hydrogen-like p-orbitals.

   (1) Find the possible states $|\,L\,S\rangle$ of the two electron system allowed by the Pauli exclusion principle. Here S is the total spin and L is the total orbital angular momentum of the two electrons. Show how you obtain your results. (8%)

   (2) If Coulomb interaction is taken into consideration, which one of the $|\,L\,S\rangle$ states has the lowest energy ? Show how you arrive at this conclusion. (7%)

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4. Photons scattering off from a particle of mass m can behave like a wave or a particle depending on the wavelength λ of the photons. If particle nature of the photons is to exhibit in the scattering,

   (1) what should be the typical photon wavelength λ_p ? (3%) Give a physical argument for your expression of λ_p . (3%)

   (2) Suppose that we have a crystal with lattice spacing $a = a_B$, where a_B is the Bohr radius, what is the expression of the Bohr radius in terms of the electron mass m_e , charge e , and Planck constant ? (3%) What typical λ_w should the photon be if it is scattered like a wave by the crystal ? (3%)

   (3) If a particle moves in the crystal with an effective mass m, and a photon incident upon the particle-crystal system, what is the largest value of m/m_e such that the photon encounters particle-like scattering from the particle and wave-like scattering from the crystal ? (3%)

5. Particle A is decayed into two particles of rest masses m_1 , m_2 , and speeds v_1 , v_2 , respectively. The velocities of the two particles are perpendicular to each other just after the decay. If relativistic effect is significant,

   (1) write down the energy equation for the decay process ; (5%)

   (2) write down the momentum equation for the decay process ; (5%)

   (3) derive the rest mass m_A of particle A ; (7%)

   (4) check your expression for m_A when $v_1 = v_2 = 0$. (3%)

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