所別：物理學系暨生物物理研究所碩士班　科目：古典物理

You must show the procedures of your calculation; otherwise you won't get any credit.

1. Let us choose the generalized coordinates to be the rectangular coordinates. Prove that the Lagrange's equations (for a single particle) can yield the Newtonian equations. 20%
2. A particle of mass m is constrained to move on the inside surface of a smooth cone of half-angle α (see Figure 1). The particle is subject to a gravitational force. (a) Determine a set of generalized coordinates and determine the constraints. (10%) (b) Find Lagrange's equation of motion. (10%) (20%)
3. Figure 2 represents a simple mechanical oscillator with the damping constant b and a series RLC circuit, respectively. Write down the equation of motion for the oscillator and Kirchhoff's equation for the electric circuit. (15%)
4. A circular loop located on $x^2 + y^2 = 9$, $z = 0$ carries a direct current of 10 A along a_ϕ. Determine H at (0, 0, 4). (Figure 3) 15% (b) A solenoid of length ℓ and radius a consists of N turns of wire carrying current I. Show that at point P along its axis, $H = \frac{nI}{2}(\cos\theta_2 - \cos\theta_1)a_z$ where $n = N/\ell$, θ_1 and θ_2 are the angles subtended at P by the end turns as illustrated by Figure 4 (hint: use part (a) can be treated as a single circular loop in the solenoid) 10% (25%)
5. The volume between two concentric conducting spherical surfaces of radii a and b (a , b) is filled with an inhomogeneous dielectric constant $\varepsilon = \frac{\varepsilon_0}{1+Kr}$, where ε_0 and K are constants and r is the radial coordinate, while the outer surface is grounded. Find: (a) The displacement in the region a < r <b. 5% (b) The capacitance of the device. 5% (c) The polarization charge density in a < r < b. 5% (d) The surface polarization charge density at r = a and r = b. 5% (20 %)

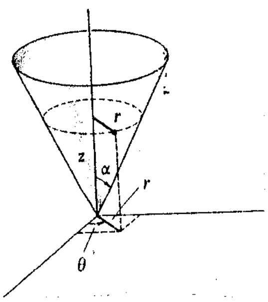

Fig.1

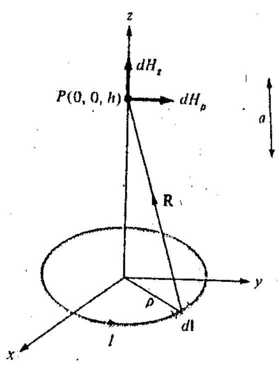

Fig.3

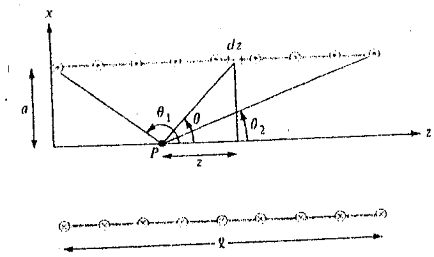

Fig.4

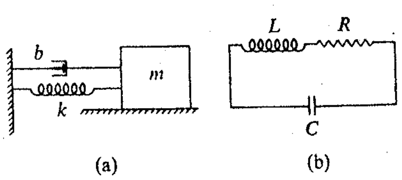

Fig. 2