1. Consider a rigid pendulum shown in figure 1. The mass of the pendulum is localized at the end and the length of the pendulum is $l$.

   (a) Plot the potential energy $U(\theta)$ of the pendulum as a function of $\theta$ for $0 \leq \theta \leq 2\pi$. (5 points)

   (b) Plot trajectories of the pendulum in the $(\theta,\ d\theta/dt)$ plane. For (i) low energy such that the maximum magnitude of $\theta$ is less than $\pi$, and (ii) high energy such that the magnitude of $d\theta/dt$ is nonzero at $\theta = \pi$. $\theta$ is allowed to be greater than $2\pi$ (5 points)

   (c) Find the lowest order approximation for the period of the motion of case (i) in (b) with very low total energy, and point out what type of motion is it. Let $\theta(t = 0) = \theta_0$ and $d\theta/dt = 0$ at $t = 0$. (5 points)

   (d) Find the lowest order approximation for the period of the motion of case (ii) in (b) with very high total energy, and point out what type of motion is it. Let $\theta(t = 0) = 0$ and $d\theta/dt = \dot{\theta}_0$ at $t = 0$. (5 points)

2. Consider $N$ gas molecules in a cubic box of lateral size $L$ at temperature $T$. Neglect the interactions between the particles

   (a) Relate the pressure in the box to the collisions between the molecules and the box (equations are not necessary). (7 points)

   (b) Derive an expression for the pressure of the box as a function of the kinetic energy of the molecules. (7 points)

   (c) Given that energy equipartition holds for the kinetic energy of the particles, relate the pressure of the box with $N$, $T$ and $L$. (6 points)

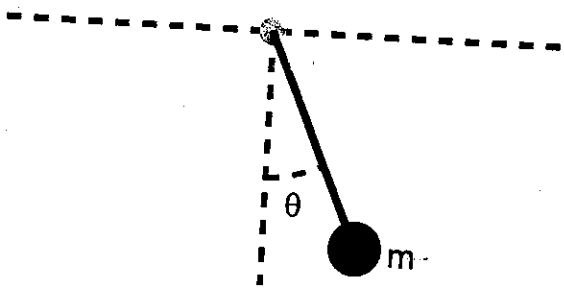


Figure 1:

3. Wave-particle duality is a puzzle in the early days of quantum mechanics.

   (a) Describe an experiment that demonstrates that electrons are particles (7 points)

   (b) Describe an experiment that demonstrates that electrons are waves (7 points)

   (c) Discuss whether an electron is a particle or a wave. You have to explain your reason. (6 points)

4. Let us consider how do dielectrics contribute to the electric field.

   (a) Explain (with very short calculation) how does an electric dipole response to an electric field that is uniform in space. (6 points)

   (b) From (a), explain how does dielectrics contribute to electric field. Does it increase or reduce the imposed electric field? (6 points)

   (c) Consider a parallel plate capacitor with area $A$ and distance $d$. Calculate its capacitance when there is nothing between the plates, then calculate the capacitance for the case when dielectric material with permittivity $\epsilon$ is filled between the plates. (8 points)

5. Consider a small segment of a string which is under tension $T$ (figure 2). The mass density of the string is $\lambda$.

   (a) Let $\theta(x)$ be small everywhere, show that the vertical force acting on a segment $dx$ is simply $Tdx(\partial^2 y/\partial x^2)$. This also gives the instantaneous acceleration of this segment in the vertical direction, i.e., $\lambda dx(\partial^2 y/\partial t^2)$. (8 points)

   (b) Show that the string satisfies $\frac{\lambda}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$. (4 points)

   (c) Show that any $y(x,t) = A(x-vt)$ is a solution of the wave equation in (b), where $A$ is an arbitrary constant and $v = \sqrt{T/\lambda}$. (8 points)

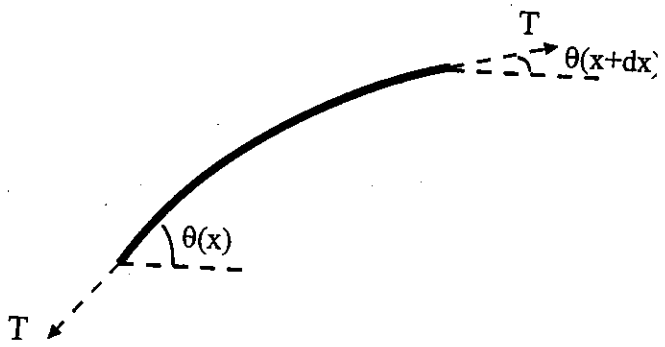


Figure 2: