

1. (a) (12 points) Use Laplace transform to solve the given equation.

$$\frac{d^2x(t)}{dt^2} + 9x(t) = f(t), x(0) = 0, x'(0) = 3, \text{ where } f(t) = \begin{cases} \cos 3t, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases}$$

(b) (13 points) Please simplify the diagram in Figure 1 and find the transfer function of $Y(s)/X(s)$.

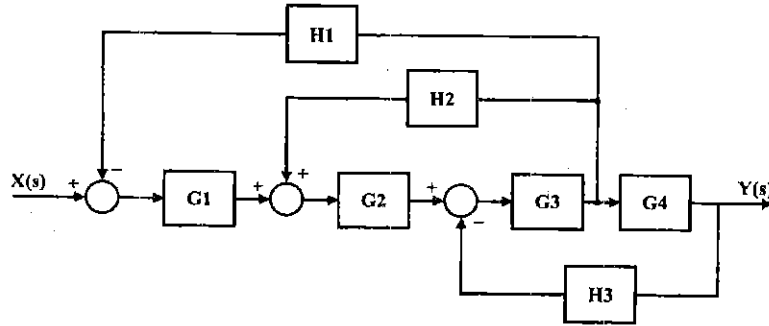


Figure 1.

參考用

2. A block diagram of a space vehicle control system is shown in the Figure 2.

(a) (13 points) Determine the gain K such that the phase margin is 50° .

(b) (12 points) What is the gain margin in this case?

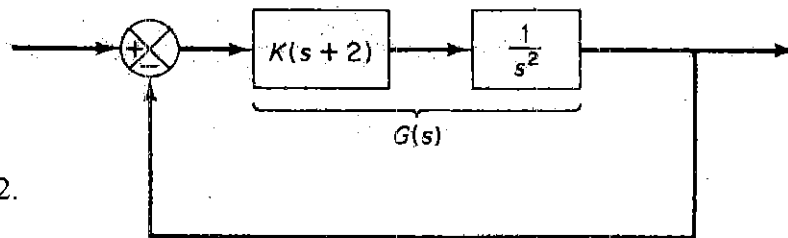


Figure 2.

3. Consider a system with an unstable plant as shown in Figure 3. Using the root-locus approach, design a proportional-plus-derivative controller (PD controller) for the closed system. The parameters K_p, T_d and T_i are parameters of the controller (represented by $C(s)$).

(a) (5 points) Among K_p, T_d, T_i , which parameter in $C(s)$ is zero for a PD controller?

(b) (5 points) Please derive the closed loop transfer function from $R(s)$ to $Y(s)$ (The transfer function contains the parameters K_p, T_d, T_i).

(c) (5 points) Find the region of K_p (in term of T_d and T_i) for the PD controller to stabilize the closed loop system.

(d) (10 points) Consider the transfer function in (b), if we wish the damping ratio (usually denoted by ζ) of the closed-loop system equal to $1/\sqrt{2}$ and the undamped natural frequency (usually denoted by ω_n) equal to 2, please determine the values for the parameters of the PD controller.

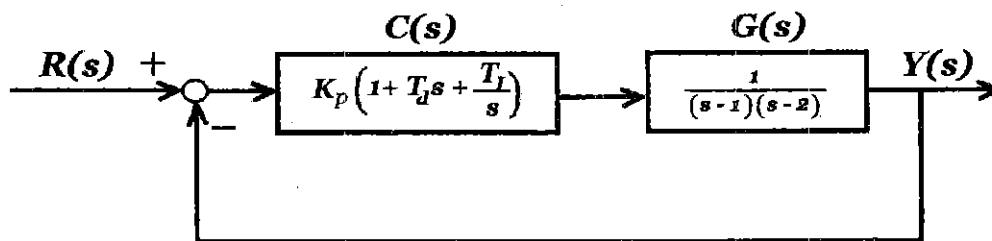


Figure 3.

4. Consider the following system as shown in the Figure 4.

(a) (13 points) If $K = 6$, find the value of α to make the system Type 1.

(b) (12 points) Find the corresponding velocity constant K_v .

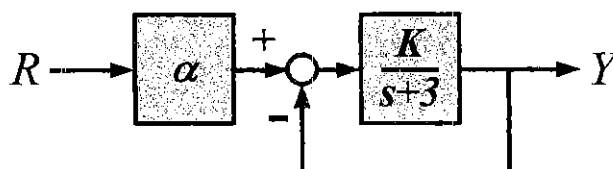


Figure 4.