科目：近代物理(3003)

校系所組：中央大學照明與顯示科技研究所
交通大學電子物理學系（丙組）
交通大學物理研究所
清華大學物理學系
清華大學先進光源科技學位學程
陽明大學生醫光電研究所（理工組）

Please box your final answer!

1. (one-dimensional simple harmonic oscillation, 20 points) Let us quantum mechanically consider a particle of mass m in a one-dimensional simple harmonic oscillation (SHO) with classical natural frequency ω. The Schrodinger equation of the SHO system is written as

$$H\varphi_n = \left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}m\omega^2x^2\right]\varphi_n = E_n\varphi_n, n=0,1,2,...$$

The normalized wave function of the ground state of the system is known as $\varphi_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp(-\frac{m\omega}{2\hbar}x^2)$. At time $t \ge t_0$, the potential of the SHO is abruptly changed so that the natural frequency of the SHO becomes $\omega' = 3\omega$. (a) Show the wave function of the ground state of the new SHO system. (b) What is the probability that a measurement of the energy returns the value $\hbar\omega/2$? (c)What is the probability that a measurement of the energy at $t=t_0$ would return the value $3\hbar\omega/2$? (A useful formulation for this problem:

$$\int_0^\infty \exp(-y^2)dy = \frac{\sqrt{\pi}}{2})$$

2. (three-dimensional infinite potential well, 15 points) Consider a particle of mass m confined in a "quantum box" with the following infinite confining potential:

$V(x,y,z)=V_x(x)+V_y(y)+V_z(z)$,

where $V_\alpha(\alpha) \equiv \begin{cases} 0, & 0<\alpha<L_\alpha \\ \infty, & \text{otherwise} \end{cases}$, $\alpha = x,y,z$; L_α is the side length of the box.

Let us consider the "particle in a box" problem in the Cartesian coordinates and label the eigen states and the eigen energies as $|n_x,n_y,n_z\rangle$ and E_{n_x,n_y,n_z}, respectively, where the quantum number $n_\alpha = 1,2,3...$ corresponds to the confinement in the α – directions and increases in order of increasing energy.

(a) Find the eigen energy of the state $|n_x=2,n_y=1,n_z=2\rangle$ for a *cubic* box with

$L_x = L_y = L_z \equiv a$

(b) Find the degeneracy g_s of the electronic shell of the energy $E_{n_x=2,n_y=1,n_z=2}$ for the

注意：背面有試題

科目：近代物理(3003)

校系所組：中央大學照明與顯示科技研究所
交通大學電子物理學系（丙組）
交通大學物理研究所
清華大學物理學系
清華大學先進光源科技學位學程
陽明大學生醫光電研究所（理工組）

cubic box in (a).

(c) Consider a *rectangular* box with $L_x = L_y = a, L_z = 2a$. Find the degeneracy of the electronic shell of the energy $E_{n_x=2,n_y=1,n_z=2}$.

3. (Hydrogen atom, 15 points) Consider a state of Hydrogen atom with the energy ≈ -3.4 eV. (a) Find the degeneracy of the electronic shell of the state. (b) List the all possible values of the z-component of orbital angular momentum of the electronic shell. (c) Find the energy of the emission line in the Lyman series involving the state.

4. (variational principle, 15 points) Please use the variational principle to prove that all one-dimensional attractive interactions allow for **at least** one bound state. (Hint: An attractive potential can be defined as a potential that both vanishes at $x = \pm\infty$ and obeys $V(x) = -|V(x)|$ for all x. One commonly used trial wavefunction is $\psi_\alpha(x) = (\alpha/\pi)^{1/4} e^{-\alpha x^2/2}$.)

5. (degenerate perturbation theory, 15 points) Please calculate the lowest-order energy correction to the first excited state(s) of a three-dimensional infinite potential well, $V(x,y,z) = \begin{cases} 0, & 0 < x,y,z < a \\ \infty, & otherwise \end{cases}$, due to perturbation $H' = \begin{cases} V_0, & if\ 0 < x,y < a/2 \\ 0, & otherwise \end{cases}$

6. (WKB approximation and non-degenerate perturbation theory, 20 points) It is known that the eigenvalues for an exactly-halved simple harmonic potential, $V(x) = \begin{cases} \frac{K}{2}x^2, & \text{for } x \ge x_0 \\ \infty, & \text{for } x < x_0 \end{cases}$ where $x_0 = 0$, are $\left(n + \frac{1}{2}\right)\sqrt{\frac{K}{m}}$ with n being just odd integers

(a) When $0 \ne x_0 << (Km)^{-1/4}$, please find the first-order (in x_0) correction to the eigenvalues by the WKB approximation.

(b) By shifting the coordinate when $x_0 \ne 0$, the potential can be rewritten as $V(x) = \begin{cases} \frac{K}{2}(x + x_0)^2, & \text{for } x \ge 0 \\ \infty, & \text{for } x < 0 \end{cases}$. Now $\frac{K}{2}(2xx_0 + x_0^2)$ can be viewed as a perturbation H' to the exactly-halved potential. Please use the perturbation formula to estimate the first-order correction to the ground-state energy and compare with your answer in (a).