科目：工程數學D(5006)

校系所組：中央大學電機工程學系(電子組、系統與生醫組)
交通大學電子研究所(甲組)
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1. (10 %) Consider the following linear equation system in R:

$$\begin{cases} x+ay+3z=2 \\ x+2y+2z=3 \\ x+3y+az=a+3 \end{cases}$$

Please discuss and determine all possible values of a and find its corresponding solution set conditions.

2. Let α, β, and γ be the ordered basis of $\mathbf{P}_2(R)$, $\mathbf{P}_3(R)$, and $\mathbf{M}_{2x2}(R)$, respectively defined by

$$\alpha=\left\{1,x,\frac{1}{2}(-1+3x^2)\right\},\quad \beta=\left\{1,x,\frac{1}{2}(-1+3x^2),\frac{1}{2}(-3x+5x^3)\right\}$$

and

$$\gamma=\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix},\begin{pmatrix}2&1\\0&0\end{pmatrix},\begin{pmatrix}3&2\\1&0\end{pmatrix},\begin{pmatrix}4&3\\2&1\end{pmatrix}\right\}$$

Let $\mathbf{T}:\mathbf{P}_2(R)\rightarrow\mathbf{P}_3(R)$, and $\mathbf{U}:\mathbf{P}_3(R)\rightarrow\mathbf{M}_{2x2}(R)$ be the linear transformations respectively defined by

$$\mathbf{T}(f(x))=15xf(x)\quad\text{and}\quad\mathbf{U}(g(x))=2\begin{pmatrix}g(4)&g(3)\\g(2)&g(1)\end{pmatrix}$$

(a) (5%) Compute the matrix representation of $\mathbf{T}$ in α and β.
(b) (5%) Compute the matrix representation of $\mathbf{U}$ in β and γ.
(c) (5%) Find a basis for the null space of $\mathbf{U}$.
(d) (5%) Compute the nullity and rank of $\mathbf{UT}$.

3. Let $\mathbf{T}$ be a linear operator on the inner product space R^4 defined by $\mathbf{T}(\upsilon)=A\upsilon$, for all $\upsilon\in\mathrm{R}^4$, where

$$A=\begin{pmatrix}4&5&3&0\\0&-2&-4&0\\0&5&7&0\\2&5&7&2\end{pmatrix}$$

(a) (8%) Find the eigenvalues and the corresponding eigenspaces of $\mathbf{T}$.
(b) (4%) Determine whether or not there exists an orthonormal basis of R^4 that consists of eigenvectors of $\mathbf{T}$? Explain why.
(c) (8%) Find an orthonormal basis for the $\mathbf{T}$-cyclic subspace of R^4 generated by $\upsilon=\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$

4. (2%) (a) Solve $y'=y^2, y(0)=2$. Call the solution y_c.
(2%) (b) Solve $y'=y^2-1, y(0)=1$. Call the solution y_p.
(1%) (c) Does y_c+y_p solve $y'=y^2-1, y(0)=3$? Explain.

注意：背面有試題

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5. (8%) One solution of the equation $y''+p(t)y'+q(t)y=0$ is $(1+t)^2$, and the Wronskian of any two solutions is constant. Find the general solution of $y''+p(t)y'+q(t)y=1+t$.

6. (5%) Three solutions of a 2nd-order linear equation $L(y)=g(t)$ are

$$\psi_1=2e^{t^2}+e^t, \psi_2=te^{t^2}+e^t \text{ and } \psi_3=(1+t)e^{t^2}+e^t.$$

Find the solution of the initial problem $L(y)=g(t); y(0)=3, y'(0)=0$

7. (8%) Let y be a real function of x. Find two linearly independent Frobenius solutions of the following differential equation at $x=0$:

$$2x^2y''+x(x-3)y'+3y=0$$

8. (8%) Let x_1 and x_2 be two real functions of t. Solve x_1 and x_2 for the following system of differential equations

$$\begin{cases} x_1'=4x_1-x_2 \\ x_2'=x_1+2x_2 \end{cases}, x_1(1)=5, x_2(1)=3$$

9. (7%) Given the initial value problem, $x''+4x'+13x=f(t);\ x(0)=x'(0)=0$
   (a) (3%) Express $x(t)$ in terms of $f(t)$ and convolution.
   (b) (4%) Solve $x(t)$ for $f(t)=u(t)-u(t-1)$, where $u(t)$ denotes the unit step (or Heaviside Step) function.

10. (9%) $f(t)=\begin{cases}1, & 0<t<5 \\ 0, & 5<t<10\end{cases}$ with $f(t+10)=f(t)$ is a piecewise continuous and periodic function that satisfies $f(t)=\dfrac{[f(t^+)+f(t^-)]}{2}$, where $f(t^+)$ and $f(t^-)$ are the right-hand and left-hand limits of $f(t)$ at each discontinuity.
   (a) (3%) Find the Fourier series of $f(t)$.
   (b) (3%) Let $f(t)$ be defined for $t\geq 0$; find its Laplace transform $F(s)$ for $s>0$.
   (c) (3%) Find a particular solution for $x''+16x=f(t)$.