1. [10 %] Please determine the following statement whether it is true or false: If \( X_1, X_2, \ldots, X_k \) are random variables, then \( \text{var} \left( \sum_{i=1}^{k} a_iX_i \right) = \sum_{i=1}^{k} a_i^2 \text{var}(X_i) \). \( \text{var} \{X\} \) is the variance of the random variable \( X \). Please explain for your answer.

2. [10 %] Please determine the following statement whether it is true or false: If \( X_1, X_2, \ldots, X_k \) are random variables, then \( E \left( \sum_{i=1}^{k} a_iX_i \right) = \sum_{i=1}^{k} a_iE(X_i) \). \( E \{X\} \) is the expectation of the random variable \( X \). Please explain for your answer.

3. [10 %] Sketch the double-sided spectra of \( x_a(t) = 5\cos(4\pi t - \pi/3) \).

4. [10%] Please determine the probability of the first error occurring at the 1000\(^{th}\) transmission in a digital data transmission system where the probability of error is \( p = 10^{-6} \).

5. [10 %] A random signal has the autocorrelation function \( R(\tau) = 5 + 2\Lambda(\tau/10) \)
where \( \Lambda(x) \) is the unit-area triangular function defined as \( \Lambda(t/\tau) = \begin{cases} 1 - |t/\tau| & |t| < \tau \\ 0 & \text{otherwise} \end{cases} \).
Please determine (a) The total power. (b) The ac power.

6. [10 %] (a) \( X_1 \) and \( X_2 \) are two independent Gaussian random variables. Each has zero mean and unit variance. \( Y = X_1 + X_2 \).
(a) Please determine the mean of \( Y \).
(b) Please write the expression of the probability density function (pdf) of \( Y, f_Y(y) \).
(Gaussian pdf: \( f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(x-m\right)^2/2\sigma^2}, -\infty < x < \infty \), where \( m \) is the mean and \( \sigma^2 \) is the variance.)
7. [10 %] A signal with a waveform shown in the following figure is fed into the system with the impulse response:

\[ h(t) = \delta(t) - 3\delta(t-5) + 5\delta(t-10). \]

Please sketch the output waveform of the system output.

8. [10 %] Consider the analog signal \( x_a(t) = 3\sin(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t) \). Determine the Nyquist sampling rate for \( x_a(t) \).

9. [10 %] Please write down two modulation schemes of (a) digital modulation (b) analog modulation, respectively.

10. [10 %] Digital binary data is to be transmitted through a baseband system with \( N_0 = 10^{-3} \text{ W/Hz} \) and the received signal amplitude \( A = 20 \text{ mV} \). The average probability of error \( P_e \) is given by the approximation \( P_e \equiv \frac{e^{-z}}{2\sqrt{\pi z}} \) where \( z \) is "\( E_s \)-over-\( N_o \)". (a) If \( 10^3 \) bits per second (bps) are transmitted, what is the average probability of error \( P_e \)? (b) If \( 10^5 \) bps are transmitted, to what value must \( A \) be adjusted in order to attain the same \( P_e \) as in part (a)?