1. (30%) $\forall a,b,c,d \in N$, if a and b are relative prime and $a > b$, then prove:

$$\gcd(a^m - b^m, a^n - b^n) = a^{\gcd(m,n)} - b^{\gcd(m,n)}, 0 \le m < n.$$

(hint: Euclidean Algorithm)

2. (15%) a) Suppose $|A| = 5, |B| = 10$. Find the number of functions $f : A \to B$.

b) Suppose $|A| = 5, |B| = 10$. Find the number of 1 to 1 functions $f : A \to B$.

c) Suppose $|A| = 10, |B| = 5$. Find the number of 1 to 1 functions $f : A \to B$..

3. (20%)
   a) Find the number of subsets of $S = \{1,2,3,\ldots,10\}$ that contain the number 6 and 7.
   b) Find the number of subsets of $S = \{1,2,3,\ldots,10\}$ that contain no odd numbers.
   c) Find the number of subsets of $S = \{1,2,3,\ldots,10\}$ that contain exactly 4 numbers and one of which is 2.
   d) Find the number of subsets of $S = \{1,2,3,\ldots,10\}$ that contain exactly 5 numbers and the sum of which is even.

4. (20%) Use the definition of big-oh to prove that
$(1 \times 2) + (2 \times 3) + (3 \times 4) + \ldots + ((n-1) \times n)$ is $O(n^3)$

5. (15%) Solve the recurrence relation
   $a_n = 5a_{n-1} - 4a_{n-2}, \quad a_0 = 0, a_1 = 1.$