科目：近代物理(2003)　校系所組：中央大學光電科學與工程學系照明與顯示科技碩士班
交通大學電子物理學系（丙組）
交通大學物理研究所
清華大學物理學系
清華大學先進光源科技學位學程（物理組）
清華大學材料科學工程學系（乙組）
陽明大學生醫光電研究所（理工組）

參考用

一. If two initial protons are moving toward each other with equal speed v such that the total kinetic energy is $2m_pc^2$ in the zero-momentum reference frame, then transform to the laboratory's frame in which one proton is at rest and another proton is moving, what is the kinetic energy of the moving proton in the laboratory's frame? (10 %)

二. A spaceship of mass M is initially at rest in space, then suddenly ejects 0.01M of fuel in a very short time at a speed of $c/2$ relative to the spaceship, calculate the change in the rest mass of the system. (10 %)

三. Consider a particle of mass m moving in a potential $V(x) = -V_0\delta(x-a)$, write the (a) eigenvalues (5 %) and (b) eigenfunctions (5 %) of the bound states.

四. In the hydrogen atom for angular momentum quantum number $l = 5$,

(a) Draw the diagram illustrating the possible orientations of the angular momentum vector L (2 %)

(b) What are the possible values of n (principle quantum number) and m (magnetic quantum number)? (4 %)

(c) Compute the minimum energy (2 %)

(d) What are the values of the magnitude of the total angular momentum? (2 %)

五. A container at 300 K contains He gas (4 g · mol^{-1}) at a pressure of 10^6 Pa.

(a) To what temperature will the He gas be cooled such that the use of Boltzmann distribution is no longer appropriate? (5 %)

(b) Assume He gas of total N particles in the container of volume V, using N and V to derive the critical temperature T_c below which Bose-Einstein condensation occurs. (5 %)

Boltzmann constant $k = 1.38 \times 10^{-23}$ J/K and Planck's constant $h = 6.63 \times 10^{-34}$ J · S

注意：背面有試題

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六. Consider a system of two uncoupled harmonic oscillators, which we call the plus type and the minus type. We have the annihilation and creation operators, denoted, for the plus type, by a_+ and $a_+^\dagger$ respectively. Likewise we have a_- and $a_-^\dagger$ for the minus type oscillator. We also define the number operator N_+ and N_- as follows: $N_+ = a_+^\dagger a_+$ and $N_- = a_-^\dagger a_-$

If the angular momentum operator J_z is defined as $J_z = \frac{\hbar}{2}\left(a_+^\dagger a_+ - a_-^\dagger a_-\right)$ and the ladder operator are defined by $J_+ = \hbar a_+^\dagger a_-$ and $J_- = \hbar a_-^\dagger a_+$.

(a) Show the following commutation relations:

(i) Express $[J_z, J_\pm]$ in terms of $J_\pm$. (10 %)

(ii) Express $[J_+, J_-]$ in terms of J_z. (10 %)

(b) Prove the relation

$$J_z^2 + \frac{1}{2}(J_+J_- + J_-J_+) = J^2 = \frac{\hbar^2}{2}N\left(\frac{N}{2}+1\right)$$

where the total N is defined by $N = N_+ + N_-$. (10 %)

七. (a) The probability current $\vec{S}(\vec{r},t)$ is defined by

$$\nabla\cdot\vec{S} + \frac{\partial\rho}{\partial t} = 0$$

where $\rho = \psi^*(\vec{r},t)\psi(\vec{r},t)$. Derive the above expression for $\vec{S}$ in terms of ψ^* and ψ for the Schrödinger equation with potential $V(\vec{r})$. (10 %)

(b) For a Schrödinger equation with a spherical symmetric potential $V(r)$. Calculate the value of $[\vec{L}, V(r)]$. (10 %)