科目：近代物理(2003)

校系所組：中央大學光電科學與工程學系照明與顯示科技碩士班
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1. The Hamiltonian of an axially symmetric quantum rotator is

$$H = \frac{L_x^2 + L_y^2}{2I_1} + \alpha L_z$$

(a). What is the energy spectrum of this system? Sketch the energy levels. (5 points)

(b). Calculate $\langle l, m_1|H|l, m_2\rangle$ where $l$ is the quantum number for angular meometum operator $\vec{L}^2$, and $m_{1(2)}$ is the quantum number for $L_z$. (5 points)

(c). Use raising ($L_+ = L_x + iL_y$) or lowering ($L_- = L_x - iL_y$) operator to construct the (un-normalized) eigenfunctions of $H$ for $l = 1$, $m = -1, 0, 1$. (5 points)

Hints:
* The spherical harmonics for $l = 1$, $m = 1$ is $Y_{1,1}(\theta, \phi) = \langle\theta, \phi|l = 1, m = 1\rangle = Ae^{i\phi}\sin\theta$.
* $L_\pm = \pm\hbar e^{\pm i\phi}(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi})$.

2. Generalized uncertainty relation is

$$(\Delta\hat{A})^2(\Delta\hat{B})^2 \geq (\frac{1}{2i}\langle[\hat{A}, \hat{B}]\rangle)^2 \quad (1)$$

where the uncertainlty of the operator $\hat{A}$ is defined as: $\Delta\hat{A} \equiv \hat{A} - \langle\hat{A}\rangle$.

(a). Prove the commutator between the position operator $\hat{x}$ and momentum operator $\hat{p}$ is given by $[\hat{x}, \hat{p}] = i\hbar$. Use this result and the generalized uncertainlty relation to explain why one can not accurately measure both the position and momentum of a quantum particle at the same time. (10 points)

(b). Can one accurately measure both $\vec{L}^2$ and $L_z$ at the same time? (where $\vec{L}^2$ is the square of the angular momentum operator $\vec{L}$ and $L_z$ is the $z-$component of $\vec{L}$) Why? Or why not? Please explain your reason in terms of the commutator $[\vec{L}^2, L_z]$. (10 points)



注意：背面有試題

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3. A quantum particle in an one-dimensional infinite square well with potential $V(x) = 0$ for $-a/2 < x < a/2$ and $V(x) = \infty$ for $|x| > a/2$. The particle has an initial wave function

$$\Psi(x, t=0) = \frac{1}{\sqrt{3}}(\Psi_1(x) + \sqrt{2}\Psi_2(x)) \tag{2}$$

where $\Psi_1(x)$, $\Psi_2(x)$ are the stationary wave functions of the ground state and first excited state of the system with eigenenergy $E_1$, $E_2$, respectively.

(a). Find $\Psi_1(x)$, $\Psi_2(x)$ and their corresponding eigenenergies $E_1$ and $E_2$. (4 points)

(b). Calculate $|\Psi(x,t)|^2$, show that it oscillates in time and find out the oscillation frequency in terms of $\omega \equiv \pi^2\hbar/(2ma^2)$. (4 points)

(c). Compute $\langle x\rangle(t)$ and $\langle p\rangle(t)$. (4 points)

(d). What are the probablities of finding the particle at ground state ($P_1$) and first excited state ($P_2$)? (3 points)

4. (a) Give one example for bosons and one example for fermions. (5 points) (b) Derive the "Pauli exclusion principle" by constructing two-fermion wave functions. (10 points)

5. What are the spin singlet and triplet states for electrons, respectively? Assign the quantum number $s$ and $s_z$ to each case. (10 points)

6. (a) Express the first order correction to the energy due to perturbation. No proof is necessary. (5 points) (b) Give one example that the degeneracy is broken by perturbation. (5 points)

7. Explain briefly the following terms: (a) graphene; (b) dark energy (c) topological insulators (d) Higgs boson (e) iron-based superconductors. (3 points each)