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※請在答案卷內作答

1. (22%)

(a) (6%) Simulate an atom by the 1-D Schrodinger equation for a single electron subject to an attraction potential $V(x) = -\alpha \cdot \delta(x)$. Please solve for the ground-state eigenfunction and eigenenergy.

(b) (8%) Simulate a molecule by doubling the potential to $V(x) = -\alpha \cdot \delta(x) - \alpha \cdot \delta(x - L)$. Find all bound eigenfunctions and corresponding energies. In the limit $L \to 0$, check if the ground-state energy reduces to your answer in (a) with doubling α.

(c) (8%) Now consider two ideal (meaning non-interacting) electrons in the double potential. Taken into account that they are spin-$\frac{1}{2}$ fermions obeying the Pauli exclusion principle, write down the total (spatial and spinor) wavefunction and its corresponding energy for the ground state and first excited state(s).

2. (24%)

Explain briefly the essential physics behind the following solid-state jargons.

(a) (4%) The Weidemann-Franz law for the thermal and electric conductivities

(b) (4%) How the energy band structure of a solid determines whether the material is a conductor, an insulator, or a semiconductor.

(c) (4%) Diodes and transistors

(d) (4%) Brillouin zone and the origin of forbidden bands

(e) (4%) Type-I and II superconductors and the Meissner effect

(f) (4%) The Bardeen-Cooper-Schrieffer (BCS) theory

3. (18%)

For N-number of ideal electrons in volume V at zero temperature, derive the expression for the following properties in terms of N, V, electron mass m_e, and the Planck constant, h.

(a) (6%) Fermi energy (*Hint*: defined as the highest energy of an occupied state.)

(b) (6%) Total energy of the electron gas

(c) (6%) Quantum pressure (*Hint*: differentiate the total energy with respect to V and then multiply the result by -1.)

注意：背面有試題

參考用

4. (18%)

(a) (4%) State the two postulates of special relativity

(b) (6%) The muon mass is 105 MeV/c^2 and its mean lifetime at rest is 2.2 μs. For a mono-energetic muon beam with E_μ=210 GeV, what is the average distance the muons in the beam can travel before their decays?

(c) (8%) Explain the following terms: (1) *proper time*, (2) *Lorentz contraction*, (3) *four vector*.

5. (18%)

Let us consider the β decay of carbon-14 ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^- + \bar{\nu}_e$.

(a) (6%) Denoting the masses of carbon-14, nitrogen-14 and electron as M_C, M_N, and m_e, respectively, and assume the neutrino is massless, calculate the maximal kinetic energy of the electron in the final state.

(b) (6%) Sketch the kinetic-energy distribution of the electron.

(c) (6%) The electron anti-neutrino $\bar{\nu}_e$ in the β decay was postulated by W. Pauli in 1930. Explain Pauli's arguments for postulating the neutrino.

注意：背面有試題

參考用