Part A: 单选题，答案请填在答题卡。每一题5分，答错倒扣1分，不作答不计分亦不扣分。

Planck constant $h = 6.63 \times 10^{-34}$ J-s; speed of light $c = 3 \times 10^8$ m/s; electron charge $e = -1.60 \times 10^{-19}$ C; electron mass $m_e = 9.11 \times 10^{-31}$ kg = 511 keV/c$^2$; Bohr radius $a_0 = 5.29 \times 10^{-11}$ m;

1. The discovery that electron possesses spin is important for the explanation of which of the following topics? (A) The cyclotron motion of a moving electron in a magnetic field (B) The photoelectric effect (C) The existence of isotopes (D) The structure of periodic table (E) The existence of anti-particle

2. In the process of beta decay, a neutron decays into a proton and other light particles as shown in the right figure. What is the mediator for this interaction? (A) $\pi^-$ (B) $W^-$ (C) $\mu^-$ (D) photon (E) gluon

3. Suppose 149 g sample of $^{149}$Sm (atomic mass = 149 u) is observed to decay at a rate of 33 Bq, what is the half-life of the sample in seconds? (1 Bq = 1 decay count/sec, 1 mole = $6 \times 10^{23}$) (A) $1.26 \times 10^{22}$ (B) $3.03 \times 10^{23}$ (C) $1.82 \times 10^{22}$ (D) $5.5 \times 10^{23}$ (E) $3.03 \times 10^2$

4. Taiwan High Speed Rail travels from Taipei to Kaohsiung (distance = 300 km) at a speed of 216 km/hour. What is the difference in travel time as measured by an observer on the ground and a passenger on the train? (A) 5 ns (B) 0.1 ns (C) 0.3 ns (D) 2 ns (E) 20 ns

5. An atom at rest is originally in the excited state and then emits a photon spontaneously and returns to the ground state. The energy difference between the two states is $\Delta E$. Including the recoil effect, what is the frequency of the emitted photon? The atom has mass $m$. (A) $\frac{\Delta E}{h}$ (B) $\frac{\Delta E + mc^2}{h}$ (C) $\frac{\Delta E^2}{2hmc^2}$ (D) $\frac{\Delta E - mc^2}{2h(\Delta E)}$ (E) $\frac{\Delta E - mc^2}{h}$

6. Two protons are moving toward each other, each with speed 0.5$c$ relative to the laboratory frame. If an observer is moving at 0.5$c$ together with one of the protons, what is the kinetic energy of the other proton as measured by the observer? The rest energy of one proton is $m_pc^2 = 938$ MeV. (A) 145 MeV (B) 290 MeV (C) 375 MeV (D) 625 MeV (E) 750 MeV

7. How many photons are emitted per second from a green laser pointer with power of 1 mW and wavelength of 532 nm? (A) $2.67 \times 10^{15}$ (B) $2.67 \times 10^{18}$ (C) $5.33 \times 10^{22}$ (D) $8.02 \times 10^{23}$ (E) $1.77 \times 10^{27}$

8. The first excited state of helium has the configuration $(1s)^1(2s)^1$, in which the two electron spins are parallel. Which of the following statement is correct? (A) Total spin is 1/2. (B) This state has no Zeeman effect. (C) The term symbol is $2^3S_1$. (D) Total angular momentum is zero. (E) Total magnetic moment is zero.

9. A photon of energy $2.4 \times 10^3$ eV collides with a free electron. What is the maximum energy the electron can acquire in this collision? (A) 9 eV (B) 11 eV (C) 18 eV (D) 21 eV (E) 23 eV

注意: 背面有試題
Part B: 计算题，请详列计算过程，无计算过程不予计分。

1. (20%) de Broglie suggested that \( E = h \nu \) and \( p = h/\lambda \) for both photons and massive particles. As a result, the electron also has wave-like behavior.

(a) (10%) For an electron with mass \( m_e \) and momentum \( p \); find the phase velocity and group velocity of the electron. You need to use relativistic energy.

(b) (5%) Verify that the group velocity cannot exceed \( c \).

(c) (5%) Show that for classical limit \( p << m_e c \), the group velocity reduces to \( p/m_e \).

2. (20%) The first three low-lying electron’s wave functions of hydrogen atom are given as:

\[
\psi_{1s} = \frac{1}{\sqrt{\pi}} (a_0)^{3/2} e^{-r/2a_0}, \quad \psi_{2s} = \frac{1}{2\sqrt{2\pi}} (a_0)^{3/2} e^{-r/2a_0} \left( 1 - \frac{r}{2a_0} \right), \quad \psi_{2p} = \frac{1}{4\sqrt{2\pi}} (a_0)^{3/2} \cdot \frac{r}{a_0} \cdot e^{-r/2a_0} \cdot \cos \theta
\]

with the normalization condition:

\[
\int_0^{2\pi} \int_0^\pi \int_0^\infty \psi^* \psi(r) r^2 \sin \theta \, dr \, d\theta \, d\phi = 1.
\]

(a) What is the most likely location from the origin for the 1s electron? (in terms of \( a_0 \))

(b) What is the mean radius of the 1s electron? (in terms of \( a_0 \)) You may need this integral:

\[
\int_0^\infty x^3 e^{-x} \, dx = 3! = 6
\]

(c) Find the charge density of the electron at the origin \( r = 0 \) in unit of coulomb/m^3 for the three states, 1s, 2s, and 2p.

(d) For an electric-dipole allowed transition between state 1 and state 2, the transition probability is proportional to the transition dipole matrix element:

\[
\int \psi_1^* (r \cdot |E_0| \cos \theta) \psi_2 \, d^3r,
\]

where \( E_0 \) is the electric field of the applied laser light. Show that 1s→2s transition is forbidden while 1s→2p is allowed.

3. (15%) Consider \( N \) identical spin-1/2 fermions, each of mass \( m \). Suppose they are trapped in a two-dimensional simple harmonic potential, \( V(x, y) = \frac{1}{2} m \omega^2 (x^2 + y^2) \). The quantized energy levels are:

\[
E_n = (n_x + n_y + 1)\hbar \omega, \text{ where } n_x, n_y = 0, 1, 2, 3, \ldots
\]

For \( n = n_x + n_y = 1 \), there are four degenerate states with equal energy \( E_1 = 2\hbar \omega \). These are \((n_x, n_y) = (1, 0)\) and \((0, 1)\) and they can be in the spin-up or spin down state as shown in the figure.

(a) (4%) Write down the general expression for the number of degenerate states with energy \( E_n = (n + 1)\hbar \omega \).

(b) (7%) Sum the total number of possible states with energy \( E \leq E_n \) and set it equal to \( N \). The highest energy \( E_n \) is the Fermi energy of the system. Derive the Fermi energy when \( N \gg 1 \).

(c) (4%) At what temperature will the \( N \) particles become degenerate Fermi gas as calculated in (b)?