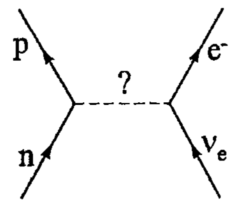
※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

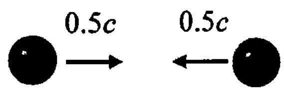
Part A: 單選題，答案請填於答案卡。一題 5 分，答錯倒扣 1 分，不作答不計分亦不扣分。

Planck constant $h = 6.63 \times 10^{-34}$ J·s; speed of light $c = 3\times10^{8}$ m/s; electron charge $e = -1.60 \times10^{-19}$ C; electron mass $m_e = 9.11 \times 10^{-31}$ kg = 511 keV/c^2; Bohr radius $a_0 = 5.29\times10^{-11}$ m;

1. The discovery that electron possesses spin is important for the explanation of which of the following topics? (A) The cyclotron motion of a moving electron in a magnetic field (B) The photoelectric effect (C) The existence of isotopes (D) The structure of periodic table (E) The existence of anti-particle
2. In the process of beta decay, a neutron decays into a proton and other light particles as shown in the right figure. What is the mediator for this interaction? (A) π^- (B) W^- (C) μ^- (D) photon (E) gluon



3. Suppose 149 g sample of ^{149}Sm (atomic mass = 149 u) is observed to decay at a rate of 33 Bq, what is the half-life of the sample in seconds? (1 Bq = 1decay count/sec, 1 mole = 6×10^{23}) (A) 1.26×10^{22} (B) 3.03×10^{23} (C) 1.82×10^{22} (D) 5.5×10^{-23} (E) 3.03×10^{-2}
4. Taiwan High Speed Rail travels from Taipei to Kaohsiung (distance = 300 km) at a speed of 216 km/hour. What is the difference in travel time as measured by an observer on the ground and a passenger on the train? (A) 5 ns (B) 0.1 ns (C) 0.3 ns (D) 2 ns (E) 20 ns
5. An atom at rest is originally in the excited state and then emits a photon spontaneously and returns to the ground state. The energy difference between the two states is ΔE. Including the recoil effect, what is the frequency of the emitted photon? The atom has mass m. (A) $\frac{\Delta E}{h}$ (B) $\frac{\Delta E + mc^2}{h}$ (C) $\frac{\Delta E}{h} - \frac{(\Delta E)^2}{2hmc^2}$ (D) $\frac{\Delta E}{h} - \frac{m^2c^4}{2h(\Delta E)}$ (E) $\frac{\Delta E - mc^2}{h}$
6. Two protons are moving toward each other, each with speed 0.5c relative to the laboratory frame. If an observer is moving at 0.5c together with one of the proton, what is the kinetic energy of the other proton as measured by the observer? The rest energy of one proton is m_pc^2 = 938 MeV. (A) 145 MeV (B) 290 MeV (C) 375 MeV (D) 625 MeV (E) 750 MeV



7. How many photons are emitted per second from a green laser pointer with power of 1 mW and wavelength of 532 nm? (A) 2.67×10^{15} (B) 2.67×10^{18} (C) 3.53×10^{22} (D) 8.02×10^{23} (E) 1.77×10^{27}
8. The first excited state of helium has the configuration $(1s)^1(2s)^1$, in which the two electron spins are parallel. Which of the following statement is correct? (A) Total spin is 1/2. (B) This state has no Zeeman effect. (C) The term symbol is 2^3S_1. (D) Total angular momentum is zero. (E) Total magnetic moment is zero.
9. A photon of energy 2.4×10^3 eV collides with a free electron. What is the maximum energy the electron can aquire in this collision? (A) 9 eV (B) 11 eV (C) 18 eV (D) 21 eV (E) 23 eV

注意:背面有試題

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Part B: 計算題，請詳列計算過程，無計算過程不予計分。

1. (20%) de Broglie suggested that $E = h\nu$ and $p = h/\lambda$ for both photons and massive particles. As a result, the electron also has wave-like behavior.
   (a) (10%) For an electron with mass m_e and momentum p, find the phase velocity and group velocity of the electron. You need to use relativistic energy.
   (b) (5%) Verify that the group velocity cannot exceed c.
   (c) (5%) Show that for classical limit $p << m_e c$, the group velocity reduces to p/m_e.

2. (20%) The first three low-lying electron's wave functions of hydrogen atom are given as:

$$\psi_{1s} = \frac{1}{\sqrt{\pi}}(a_0)^{-\frac{3}{2}} \cdot e^{-\frac{r}{a_0}},\ \psi_{2s} = \frac{1}{2\sqrt{2\pi}}(a_0)^{-\frac{3}{2}} \cdot e^{-\frac{r}{2a_0}}\left(1-\frac{r}{2a_0}\right),\ \psi_{2p} = \frac{1}{4\sqrt{2\pi}}(a_0)^{-\frac{3}{2}} \cdot \frac{r}{a_0} \cdot e^{-\frac{r}{2a_0}} \cdot \cos\theta$$

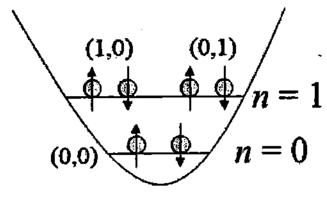
with the normalization condition: $\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{\infty} \psi^* \psi(r)\ r^2 \sin\theta\ dr\ d\theta\ d\phi = 1$.

   (a) What is the most likely location from the origin for the 1s electron? (in terms of a_0)
   (b) What is the mean radius of the 1s electron? (in terms of a_0) You may need this integral:

$$\int_0^\infty x^3\ e^{-x} dx = 3! = 6$$

   (c) Find the charge density of the electron at the origin (r = 0) in unit of coulomb/m^3 for the three states, 1s, 2s, and 2p.
   (d) For an electric-dipole allowed transition between state 1 and state 2, the transition probability is proportional to the transition dipole matrix element: $\int \psi_1^*(r \cdot |E_0| \cos\theta)\psi_2 d^3\vec{r}$, where E_0 is the electric field of the applied laser light. Show that 1s→2s transition is forbidden while 1s→2p is allowed.

3. (15%) Consider N identical spin-1/2 fermions, each of mass m. Suppose they are trapped in a two-dimensional simple harmonic potential, $V(x,y) = \frac{1}{2}m\omega^2(x^2+y^2)$. The quantized energy levels are:
   $E_n = (n_x + n_y + 1)\hbar\omega$, where n_x, n_y = 0, 1, 2, 3…
   For $n = n_x + n_y = 1$, there are four degenerate states with equal energy $E_1 = 2\hbar\omega$. These are (n_x, n_y) = (1, 0) and (0, 1) and they can be in the spin-up or spin down state as shown in the figure.



   (a) (4%) Write down the general expression for the number of degenerate states with energy $E_n = (n+1)\hbar\omega$.
   (b) (7%) Sum the total number of possible states with energy $E \le E_n$ and set it equal to N. The highest energy E_n is the Fermi energy of the system. Derive the Fermi energy when $N >> 1$.
   (c) (4%) At what temperature will the N particles become degenerate Fermi gas as calculated in (b)?