科目：天文學(2004)　　校系所組：中央大學天文研究所

1 Properties of Stars

1.1 Magnitude System

The magnitude system used in astronomy has a logarithmic scale, and 5 mag difference corresponds to the 100 times difference in brightness. Derive the Pogson's fomula

$$m_1 - m_2 = -2.5 \log \frac{F_1}{F_2},$$

where m_1, m_2, F_1, F_2 are magnitude of star 1, magnitude of star 2, brightness of star 1, and brightness of star 2, respectively. (10 points)

1.2 Distance to Stars

The most reliable method to measure the distance to stars is the measurement of the annual parallax. What is the relationship between the annual parallax p and the distance d? (Note clearly the unit you use.) (10 points)

1.3 Apparent and Absolute Magnitudes

Absolute magnitude is defined as the magnitude as seen from the distance of 10 parsec. Show that the absolute magnitude M can be expressed as

$$M = m + 5 - 5 \log d,$$

where m is the apparent manigutde and d is the distance in parsec. (10 points)

1.4 Luminosity of Stars

Combine the distance modulus $M = m + 5 - 5 \log d$ and Pogson's fomula, and show the luminosity of a Sun-like star L is written as

$$L \sim 3 \times 10^{28} \times 10^{-0.4M} \text{ [W]}.$$

Use the Sun as a reference. The luminosity and the apparent magnitude of the Sun are $L_\odot = 3.84 \times 10^{26}$ W and $m_V = -26.75$ mag, respectively. Note that 1 AU $= 1.50 \times 10^{11}$ m and 1 parsec $= 3.09 \times 10^{16}$ m. (10 points)

1.5 Surface Temperature of Stars

The surface temperature of a star can be estimated by obtaining the spectrum. Desribe the way to know the stellar surface temperature from its spectrum. (10 points)

For the convenience, we use following formula for later discussion.

$$B - V = -0.865 + \frac{8540}{T}$$

Here, $B - V$ and T are color index and surface temperature (in K) of the star, respectively.

1.6 Radius of Stars

Show that the radius of a star R can be expressed as

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}},$$

where σ is Stephan-Boltzmann constant ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}). (10 points)

1.7 Radius of Aldebaran

Following results for Aldebaran, a red giant, have obtained from observations. Calculate the radius of Aldebaran, and compare it to solar radius. (10 points)

Visual apparent magnitude	+0.85 mag
$B - V$ color index	+1.54
Annual parallax	50.1 mas (milli-arcsec)

注意：背面有試題

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2 Energy Source of Stars

2.1 Chemical Energy

- 1 kg of oil can generate 10,000 kcal of energy. How much oil is needed for every second to keep shining the Sun with the current solar luminosity? Note that 1 cal = 4.2 J. (5 points)
- How long the Sun can keep shining if the Sun is made of pure oil and the source of solar energy is the chemical energy like the energy generated by burning the oil? Compare it to the age of the Sun. (5 points)

2.2 Nuclear Energy

The mass can be converted into the energy. For a proton-proton chain I reaction, 4 hydrogen nuclei are converted into a helium-4 nucleus. The masses of a hydrogen atom, a proton, and a helium-4 nucleus are 1.674×10^{-27} kg, $m_p = 1.6726 \times 10^{-27}$ kg, and $m_{He4} = 6.643 \times 10^{-27}$ kg, respectively.

- How much energy is generated by a single proton-proton chain I reaction? (5 points)
- What is the proton-proton chain I reaction rate to maintain the luminosity of the Sun? (5 points)
- How many hydrogen atoms are there in the Sun, if the Sun is purely made of hydrogen atoms? (5 points)
- How long the Sun can shine, if the source of solar energy is nuclear energy? Compare it with the age of the Sun. (5 points)

Constants

Speed of light	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron volt	$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
Radiation constant	$a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Atomic mass unit	$m_H = 1.66 \times 10^{-27}\,\mathrm{kg}$
electron mass	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
proton mass	$m_p = 1.6726 \times 10^{-27}\,\mathrm{kg}$
neutron mass	$m_n = 1.6749 \times 10^{-27}\,\mathrm{kg}$
helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27}\,\mathrm{kg}$
hydrogen atom mass	$1.674 \times 10^{-27}\,\mathrm{kg}$
helium-3 atom mass	$5.009 \times 10^{-27}\,\mathrm{kg}$
helium-4 atom mass	$6.648 \times 10^{-27}\,\mathrm{kg}$
ideal gas constant	$\mathcal{R} = 8.31 \times 10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Solar mass	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot = 6.96 \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$
Earth mass	$M_\oplus = 5.98 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus = 6.38 \times 10^{6}\,\mathrm{m}$
Astronomical unit	$1\,\mathrm{AU} = 1.50 \times 10^{11}\,\mathrm{m}$
π	$\pi = 3.14$
cal and J	$1\,\mathrm{cal} = 4.2\,\mathrm{J}$