

類組：電機類 科目：工程數學 C(3005)

※請在答案卡內作答

- 本測驗試題為多選題，請選出所有正確的答案，並請用2B鉛筆作答於答案卡。
- 共二十題，每題五分，每答對一個選項，可得一分，每答錯一個選項，倒扣一分，直到本科分數扣完為止。未作答則該題不給分也不扣分。

Notation: In the following questions, boldface lowercase letters such as \mathbf{x} , \mathbf{v} , etc. denote column vectors of proper length; boldface uppercase letters such as \mathbf{A} , \mathbf{B} , etc. denote matrices of proper size; \mathbf{A}^T means the transpose of matrix \mathbf{A} . $C(\mathbf{A})$ is the column space of matrix \mathbf{A} , and $N(\mathbf{A})$ is the null space of matrix \mathbf{A} . \mathbb{R} is the usual set of all real numbers.

一、Forward elimination uses possible elementary row operations to do elimination on a matrix. Suppose a block

lower triangular matrix $\mathbf{E} = \begin{bmatrix} \mathbf{H} & \mathbf{I} \\ \mathbf{J} & \mathbf{K} \end{bmatrix}$ can do elimination on a whole block column of matrix

$\mathbf{A} = \begin{bmatrix} \mathbf{M} & \mathbf{N} \\ \mathbf{P} & \mathbf{Q} \end{bmatrix}$ to produce an upper triangular matrix $\mathbf{U} = \begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{S} & \mathbf{T} \end{bmatrix}$. If \mathbf{H} and \mathbf{K} are identity matrices, and \mathbf{A}

has independent columns, which of the following statements are true?

- (A) $\mathbf{A}^T \mathbf{A}$ is invertible.
- (B) $\mathbf{T} = \mathbf{Q} - \mathbf{P} \mathbf{M}^{-1} \mathbf{N}$.
- (C) $\mathbf{J} = \mathbf{P} \mathbf{M}^{-1}$.
- (D) $\mathbf{Q} = \mathbf{M}$; $\mathbf{R} = -\mathbf{N}$.
- (E) \mathbf{E} is invertible.

二、Given three vectors, $\mathbf{v}_1 = [1 \ 2 \ 3 \ 4 \ 5]^T$, $\mathbf{v}_2 = [6 \ 7 \ 8 \ 9 \ 10]^T$, and $\mathbf{v}_3 = [2 \ -3 \ 0 \ 1 \ 0]^T$,

which of the following statements are true?

- (A) The set of all linear combinations of \mathbf{v}_1 , \mathbf{v}_2 and \mathbf{v}_3 is isomorphic to \mathbb{R}^3 .
- (B) If S is a subspace spanned by \mathbf{v}_1 and \mathbf{v}_2 , \mathbf{v}_3 can be a vector in one of the bases for S^\perp , which is perpendicular to S .
- (C) If $\mathbf{A} = [\mathbf{v}_1 \ \mathbf{v}_2]$, the rank of \mathbf{A} is 3.
- (D) If $\mathbf{A} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3]$, the dimension of $N(\mathbf{A})$ is 2.
- (E) If $\mathbf{A} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3]$, the column vectors of \mathbf{A}^T are linearly independent.

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三、 S_1 is the subspace generated by $\mathbf{v}_1 = [1 \ -1 \ 1 \ 1]^T$, and S_2 is the subspace generated by

$\mathbf{v}_2 = [1 \ 0 \ -1 \ 0]^T$. If $\mathbf{A} = [\mathbf{v}_1 \ \mathbf{v}_2]$, which of the following is within the subspace generated by \mathbf{v}_1 and \mathbf{v}_2 ?

(A) $S_1 \cap S_2$.

(B) $S_1 \cup S_2$.

(C) $C(\mathbf{A})$.

(D) $[4 \ -4 \ 3 \ 4]^T + c_1 * [1 \ -1 \ 1 \ 1]^T + c_2 * [1 \ 0 \ -1 \ 0]^T$, where c_1 and c_2 are arbitrary real numbers.

(E) $[x_1 \ x_2 \ x_3 \ x_4]^T$, where $\sum_{i=1}^4 x_i = 0$.

四、Given a subspace $W: ax + by + cz + du = 0$, $\{a, b, c, d \in \mathbb{R}^1\}$; If the orthogonal projection matrix onto the

W is \mathbf{P} , which of the following statements are always true?

(A) $\mathbf{P}^T = \mathbf{P}$.

(B) $\mathbf{P} = \mathbf{P}^{-1}$.

(C) The column vectors of \mathbf{P}^T are orthogonal to each other.

(D) $\mathbf{P}^2 = \mathbf{P}$.

(E) $\mathbf{P}^T \mathbf{P}$ is invertible.

五、 \mathbf{A} is an m by n matrix and $\mathbf{Q} = [\mathbf{q}_1 \ \dots \ \mathbf{q}_n]$, suppose \mathbf{Q} has orthonormal columns, i.e. \mathbf{q}_i ($i=1 \sim n$) are

orthonormal vectors, and $\mathbf{A} = \mathbf{Q}\mathbf{R}$, which of the following statements are true?

(A) $\mathbf{Q}^T = \mathbf{Q}^{-1}$.

(B) The orthogonal projection matrix onto $C(\mathbf{Q})$ is Identity Matrix.

(C) $\mathbf{q}_i^T \mathbf{q}_j = 0$ if $i \neq j$.

(D) $\det(\mathbf{Q}^T \mathbf{Q}) = 1$.

(E) If $m=n$, $\det(\mathbf{A}) = \det(\mathbf{R})$.

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六、For matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$, which of the following statements are true for each basis and dimension of

four fundamental subspaces? Note the coefficients $a, b, c \in \mathbb{R}^1$.

(A) Column space of matrix A: $C(A) = a \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, dimension $C(A) = 1$.

(B) Null space of matrix A: $N(A) = b \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, dimension $N(A) = 1$.

(C) Column space of matrix A^T : $C(A^T) = c \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, dimension $C(A^T) = 1$.

(D) Matrix A has 2 eigenvalues and eigenvectors.

(E) Rank of matrix A is 2.

七、Which of the following statements are true?

(A) A real matrix with real eigenvalues and eigenvectors is symmetric.

(B) A real matrix with real eigenvalues and complete set of orthogonal eigenvectors is symmetric.

(C) The inverse of a symmetric matrix is symmetric.

(D) If the columns of a square matrix S are linearly independent, S is invertible.

(E) A matrix with complete set of independent eigenvectors is diagonalizable.

八、Find the eigenvalues and eigenvectors of A^3 , where

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

(A) $\lambda_1 = 1, \lambda_2 = 3$; $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(B) $\lambda_1 = -1, \lambda_2 = 3$; $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(C) $\lambda_1 = 1, \lambda_2 = 9$; $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(D) $\lambda_1 = 1, \lambda_2 = 27$; $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(E) $\lambda_1 = 1, \lambda_2 = 1$; $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

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九、The rabbit and wolf population shows fast growth of rabbits (from $6r$) but loss to wolves (from $-2w$)

$$r' = 6r - 2w$$

$$w' = 2r + w$$

Which of the following statements are true?

- (A) The eigenvalues are $\lambda_1 = 5$, $\lambda_2 = 2$.
- (B) If $r(0) = w(0) = 30$, the population of rabbit at time t becomes: $20e^{5t} + 10e^{2t}$.
- (C) If $r(0) = w(0) = 30$, the population of wolf at time t becomes: $10e^{-2t} + 20e^{-5t}$.
- (D) After a long time, the ratio of wolves to rabbits approaches 2.
- (E) After a long time, the wolves will be extinct due to lack of food.

十、For a real matrix $A = \begin{bmatrix} 2 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 2 \end{bmatrix}$, which of the following statements are true?

- (A) A is positive definite if $b = 3$.
- (B) The eigenvalues in positive definite matrix must be positive.
- (C) A equals $R^T R$ for a matrix R with independent columns if $b = 0$.
- (D) All 3 pivots in matrix A are positive if $b = 0$.
- (E) The determinant of A is always positive for all b .

十一、Suppose that $y_1(x), \dots, y_n(x)$ are $n-1$ times differentiable functions over $(-\infty, \infty)$ and $W(x)$

denotes the Wronskian of $y_1(x), \dots, y_n(x)$ at x . Which of the following statements are true?

- (A) $W(x)$ vanishes at every x if $y_1(x), \dots, y_n(x)$ are linearly dependent.
- (B) If $y_1(x), \dots, y_n(x)$ are linearly independent, then there is x such that $W(x) \neq 0$.
- (C) If $y_1(x), \dots, y_n(x)$ are linearly independent, then $W(x) \neq 0$ for all x .
- (D) If $y_1(x), \dots, y_n(x)$ are also solutions of an n th-order linear homogeneous ordinary differential equation with constant coefficients. Then either $W(x) \neq 0$ for all x or $W(x) = 0$ for all x .
- (E) None of the above statements are true.

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十二、 Suppose that $p(x)$, $q(x)$, and $f(x)$ are continuous functions over $(-\infty, \infty)$, and $f(x)$ is not the zero function. Please determine which of the solution sets of the following differential equations are vector spaces?

(A) $y''(x) + p(x)y'(x) + q(x)y(x) = f(x)$.

(B) $y''(x) + p(x)y'(x) + q(x)y(x) = 0$.

(C) $y(x)y''(x) + y(x) = 0$.

(D) $x^2y''(x) + 5xy'(x) + 12y(x) = 1$.

(E) $(y'(x))^3 = 1$.

十三、 Consider the differential equation $y''(x) + ay'(x) + by(x) = 0$. Which of the following statements are true?

(A) $y(x) \rightarrow 0$ no matter what $y(0)$ and $y'(0)$ if $a > 0$ and $b > 0$.

(B) $y(x)$ is bounded no matter what $y(0)$ and $y'(0)$ if $a > 0$ and $b = 0$.

(C) $y(x)$ is unbounded for all $(y(0), y'(0)) \neq (0, 0)$ if $\min\{a, b\} < 0$.

(D) $y(x)$ is always unbounded whenever $(y(0), y'(0)) \neq (0, 0)$ if $ab < 0$.

(E) None of the above statements are true.

十四、 Which of the following statements are true?

(A) $x(t) = \int_0^t \tau e^{-\tau} f(t-\tau) d\tau$ is the solution of $x''(t) + 2x'(t) + x(t) = f(t)$ and $x(0) = x'(0) = 0$.

(B) Let $\delta(x)$ be the impulse function. Then the following two initial value problems

$$y''(x) + ay'(x) + by(x) = f(x), y(0) = 0 \text{ \& } y'(0) = v \text{ and}$$

$$y''(x) + ay'(x) + by(x) = f(x) + v\delta(x), y(0) = 0 \text{ \& } y'(0) = 0$$
 have the same solution for $x > 0$.

(C) If $f(x)$ is a continuous function over $[0, \infty)$, then there always exists some complex number s such that the Laplace transform $\int_0^{\infty} e^{-sx} f(x) dx$ converges.

(D) Inverse Laplace transformation, provided it exists, is not a linear function.

(E) None of the above statements are true.

十五、 Consider the initial value problem: $y'(x) = x^2y(x)(1-y(x))^3$, $y(0) = 0.5$. Which of the following statements are true?

(A) The initial value problem may have infinite many solutions.

(B) The solution, if exists, always lies between 0 and 1.

(C) The given differentiable equation is separable.

(D) The given differentiable equation is exact.

(E) None of the above statements are true.

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十六、 In the following choices of (A) to (E), please find the solutions to the given system of equations

$$\frac{d}{dt}\mathbf{x} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} e^t \\ \sqrt{3}e^{-t} \end{bmatrix}$$

(A) $5 \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} e^{-2t} + \begin{bmatrix} 2/3 \\ 1/\sqrt{3} \end{bmatrix} e^t - \begin{bmatrix} -1 \\ 2/\sqrt{3} \end{bmatrix} e^{-t}$

(B) $3 \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} e^{-2t} - \begin{bmatrix} 2/3 \\ 1/\sqrt{3} \end{bmatrix} e^t + \begin{bmatrix} -1 \\ 2/\sqrt{3} \end{bmatrix} e^{-t}$

(C) $5 \begin{bmatrix} -\sqrt{3} \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} e^{-2t} + \begin{bmatrix} 2/3 \\ -1/\sqrt{3} \end{bmatrix} e^t - \begin{bmatrix} 1 \\ 2/\sqrt{3} \end{bmatrix} e^{-t}$

(D) $5 \begin{bmatrix} -\sqrt{3} \\ 1 \end{bmatrix} e^{2t} + 3 \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} e^{-2t} - \begin{bmatrix} 2/3 \\ 1/\sqrt{3} \end{bmatrix} e^t + \begin{bmatrix} -1 \\ 2/\sqrt{3} \end{bmatrix} e^{-t}$

(E) $5 \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} e^{2t} + 3 \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} e^{-2t} - \begin{bmatrix} 2/3 \\ 1/\sqrt{3} \end{bmatrix} e^t + \begin{bmatrix} -1 \\ 2/\sqrt{3} \end{bmatrix} e^{-t}$

十七、 We can derive the solution as a series of normalized eigenfunctions of the corresponding homogeneous problem as follows:

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(1) + y'(1) = 0$$

where the solution can be expressed as

$$y = \sum_{n=1}^{\infty} \frac{Cf(x) \sin \sqrt{\lambda_n}}{\lambda_n (\lambda_n - 2)(1 + \cos^2 \sqrt{\lambda_n})}$$

Please find the corresponding term of $Cf(x)$ from the following choices:

(A) $4(\sin \sqrt{\lambda_n} x + \cos \sqrt{\lambda_n} x)$

(B) $4 \sin \sqrt{\lambda_n} x + \cos \sqrt{\lambda_n} x$

(C) $\sin \sqrt{\lambda_n} x + 4 \cos \sqrt{\lambda_n} x$

(D) $4 \sin \sqrt{\lambda_n} x$

(E) $2 \cos \sqrt{\lambda_n} x$

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十八、 Continuing on the previous question, please find the value of the λ_n for $n \geq 4$.

(A) $\frac{(2n+1)^2 \pi^2}{4}$

(B) $\frac{(n\pi)^2}{2}$

(C) $\frac{(n\pi+1)^2}{2}$

(D) $\frac{(2n-1)^2 \pi^2}{4}$

(E) $\frac{(n\pi-1)^2}{4}$

十九、 Find the inverse Laplace transform of the given functions:

$$L^{-1} \left\{ \frac{2s+1}{4s^2+4s+5} \right\}$$

(A) $\frac{1}{4} e^{t/2} \sinh(t-2)$

(B) $\frac{1}{2} e^{-t/2} \sin t$

(C) $\frac{1}{2} e^{-t/2} \cos t$

(D) $\frac{1}{4} e^{t/2} \cosh(t-2)$

(E) $\frac{1}{4} e^{-t/2} \sinh t$

二十、 Assume that there is a Fourier series converging to the function f defined by

$$f(x) = \begin{cases} -x, & -2 \leq x < 0, \\ x, & 0 \leq x < 2; \end{cases} \quad \text{and} \quad f(x+4) = f(x).$$

Please find the following choices, which are the terms in this Fourier series?

(A) $\frac{-8}{\pi^2} \frac{\cos(\frac{15}{2} \pi x)}{225}$

(B) $\frac{1}{\pi^2} \frac{\sin(4\pi x)}{8}$

(C) 1

(D) $\frac{8}{\pi^2} \frac{\cos(\frac{5}{2} \pi x)}{25}$

(E) $\frac{1}{\pi^2} \frac{\sin(8\pi x)}{32}$