類組：電機類　科目：工程數學D(3006)　　共 3 頁 第 1 頁

※請在答案卷內作答

1. (15 points) Let **Ax**=**b**, where

$$\mathbf{A}=\begin{bmatrix} 1 & 2 & -1 & 2 & 1 \\ -1 & -2 & 1 & 2 & 3 \\ 2 & 4 & -3 & 2 & 0 \\ -3 & -6 & 2 & 0 & 3 \end{bmatrix} \quad \textit{and} \quad \mathbf{b}=\begin{bmatrix} 2 \\ 6 \\ 3 \\ 9 \end{bmatrix}$$

   (a) (7 points) Obtain the reduced row echelon form (RREF) of [**A b**].

   (b) (2 points) Obtain the rank and nullity of **A**.

   (c) (2 points) Are the columns of **A** linearly independent? Explain your answer.

   (d) (4 points) How many solutions for **Ax**=**b**? If it has solutions, write down the general solutions.

2. (15 points) Given the following matrix:

$$\mathbf{A}=\begin{bmatrix} 2 & -1 & 3 & 2 & 1 \\ -2 & 2 & -1 & 1 & 4 \\ 4 & 1 & 15 & 12 & 19 \\ 6 & -6 & 9 & -4 & 0 \\ 4 & -2 & 9 & 2 & 9 \end{bmatrix}$$

   (a) (7 points) Obtain the determinant of **A**.

   (b) (2 points) Is **A** invertible? Explain your answer.

   (c) (4 points) Obtain the determinant of the following matrix.

$$\mathbf{B}=\begin{bmatrix} 4 & 1 & 15 & 12 & 19 \\ -2 & 2 & -1 & 1 & 4 \\ 2 & -1 & 3 & 2 & 1 \\ 4 & -2 & 9 & 2 & 9 \\ 6 & -6 & 9 & -4 & 0 \end{bmatrix}$$

   (d) (2 points) Obtain the determinant of the inverse of **B**

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參考用

3. (20 points) Given the following vector set

$$\mathrm{B} = \{\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right\}$$

(a) (8 points) Use the Gram-Schmidt process to generate a new orthonormal basis $\mathbf{V}=\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ from **B** (Please use the order $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ to generate the new basis **V**).

(b) (4 points) Use the new basis **V** to represent the following vector **u**:

$$\mathbf{u} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

(c) (4 points) Let **w** be the orthogonal projection of $\mathbf{b}_3$ to the plane spanned by $\{\mathbf{b}_1, \mathbf{b}_2\}$. Find **w**.

(d) (4 points) Find the orthogonal complement of $\{\mathbf{b}_1, \mathbf{b}_2\}$.

4. (16 points) Let $f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ x^2 & 0 \le x < \pi. \end{cases}$

(a) (8 points) Find the Fourier series of $f(x)$ on the given interval.

(b) (8 points) Use the result you obtained in part (a) to show that

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \quad \text{and} \quad \frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

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5. (10 points)

   (a) (5 points) Suppose that $y = 1 - x + 6x^2 + 3e^x + 5xe^x$ is a solution of a homogeneous fifth-order linear differential equation with constant coefficients. What is the corresponding auxiliary (or characteristic) equation of the differential equation?

   (b) (5 points) Please explain why the function $F(s) = \frac{s^2+5s+7}{s^2+2s+4}$ is not the Laplace transform of a function that is piecewise continuous and of exponential order.

6. (24 points) Solve the following differential equations:

   (a) (8 points) $y'(t) = 1 - \sin t - \int_0^t y(\tau)d\tau, \quad y(0) = 0.$

   (b) (8 points) $\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -2t^2 \\ t+5 \end{pmatrix}$

   (c) (8 points) $(e^y + 1)^2 e^{-y}dx + (e^x + 1)^3 e^{-x}dy = 0.$