

一、(10%)

The output $y[n]$ of the three-point moving average system is related to the input $x[n]$ according to the

$$\text{formula } y[n] = \frac{1}{3} \sum_{k=0}^2 x[n-k]$$

(一) (5%) Please determine the impulse response of this system

(二) (5%) Determine the output of the system when the input is the rectangular function defined as $x[n] = u[n] - u[n-5]$.

二、(15%)

(一) (7%) suppose that an LTI system has the following output $y(t)$ when the input is the unit step $x(t) = u(t)$:

$$y(t) = e^{-t}u(t) + u(-1-t)$$

determine the response of the system to the input $x(t)$ shown in Figure 1.

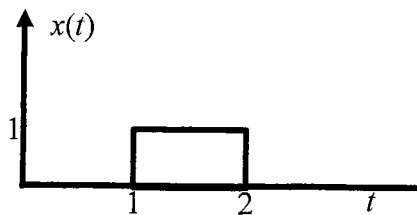


Figure 1

(二) (8%) For a discrete time LTI system: $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n]$

(1) (4%) Determine the frequency response of system.

(2) (4%) Determine the time domain impulse response of system.

三、(15%)

(一) (8%) Determine the Fourier series coefficients of $x(t) = (\sin(t))(\cos(3t))$.

(二) (7%) Given the discrete-time Fourier series coefficients $X[k] = \cos\left(\frac{3\pi}{16}k\right) + j \sin\left(\frac{5\pi}{12}k\right)$,

determine the discrete-time periodic signal $x[n]$.

四、(15%)

(一) (5%) Prove the Parseval relationship: $\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$.

(二) (5%) Determine the Fourier transform of $x(t) = e^{-2|t|}$.

(三) (5%) Let $X(e^{j\Omega})$ be the discrete-time Fourier transform of a real signal $x[n]$. Given

$$Y(e^{j\Omega}) = X(e^{j\Omega})e^{j2\Omega} + X(e^{-j\Omega})e^{-j2\Omega}, \text{ determine the discrete-time signal } y[n], \text{ expressed by } x[n].$$

五、(5%)

Given two continuous-time periodic signals $a(t)$ and $b(t)$, is the following signal: $a(t)e^{b(t)}$ always a periodic signal? Justify your answer.

六、(15%)

Given a real signal $x(t)$ which has a spectrum in Figure 2(a) with $\omega_1 > \omega_2 - \omega_1$, consider the sampling system in Figure 2(b), where $\delta(t)$ denotes the continuous-time unit impulse function and $H(j\omega)$ indicates a filter to be determined by you. Find the maximum value of T such that $x_r(t) = x(t)$. Does your answer violate the sampling theorem? Note that answers without justification get no credit.

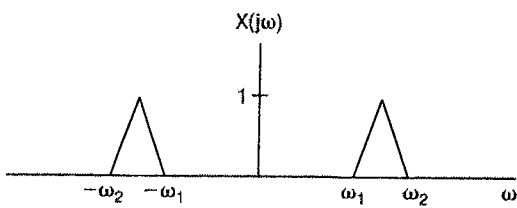


Figure 2(a)

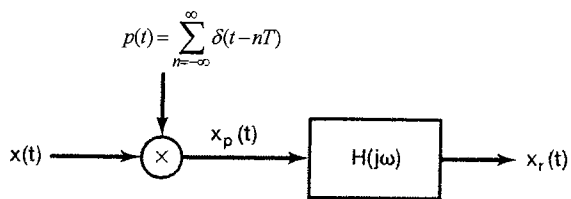


Figure 2(b)

七、(10%)

Consider a continuous-time stable and causal system with impulse response $h(t)$ and system function $H(s)$. Suppose $H(s)$ is rational, has a pole at $s = -4 + 4j$, and does not have a zero at the origin (at the origin, $s = 0 + 0j$). The location of all the other poles and zeros is unknown. For each of the following statements, please determine it is “true” or “false” and justify your decision.

(一) (4%) $F\{h(t)e^{4t}\}$ converges ($F\{\}$ represents Fourier transform)

(二) (3%) $\int_{-\infty}^{\infty} h(t)dt = 0$

(三) (3%) $H(s) = H(-s)$

八、(5%)

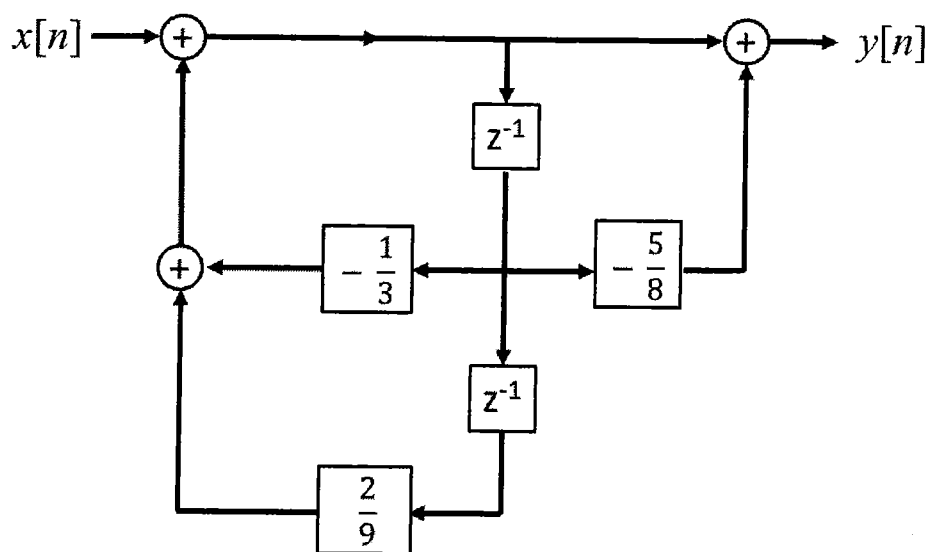
Consider a discrete-time noncausal seven-point weighted moving average system as follows.

$$y[n] = \sum_{k=-3}^3 3 \left(\frac{a}{3} \right)^{|k|} x[n-k],$$

where $x[n]$ is the input, $y[n]$ is the output, $|k|$ represents the absolute value of the integer k , and a is real. Find the system function of its causal version. Note that the causal seven-point weighted moving average system should have the same magnitude response as the noncausal one has, and should have minimum group delay when being compared with all the other causal versions.

九、(10%)

The input $x[n]$ and output $y[n]$ of a causal linear time invariant system are related through the block-diagram representation shown in the following figure.



(一) (5%) Determine a difference equation relating $y[n]$ and $x[n]$.

(二) (5%) Is this system stable?