一、(15%)

Consider a discrete-time system with input $x[n]$ and output $y[n]$. Show that the system is linear and time-invariant if and only if there exists a signal $h[n]$ such that $y[n] = x[n] * h[n]$ for all possible pairs of $x[n]$ and $y[n]$, where * denotes the operator taking convolution.

二、(10%)

(一) (5%) Consider a continuous-time linear and time-invariant (LTI) system with input $x(t)$ and output $y(t)$. Given the frequency response $H(j\omega)$ of the system, express the Fourier transform of $y(t)$ in terms of $H(j\omega)$ and the Fourier transform of $x(t)$. Prove your answer.

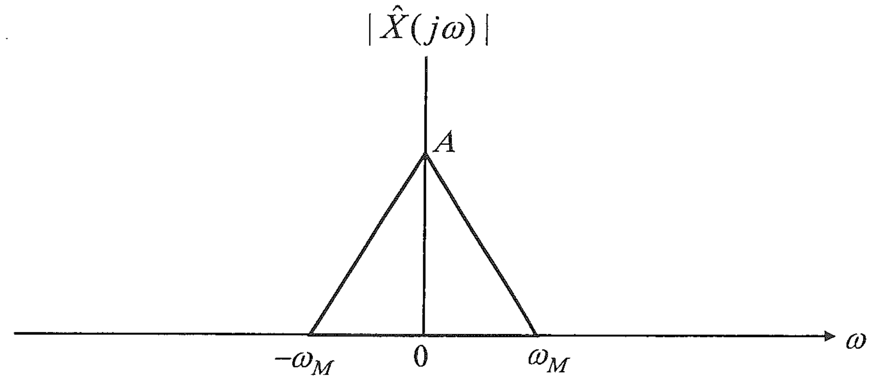
(二) (5%) An LTI system is said to have phase lead at a particular frequency $\omega = \omega_0$ if, for the input $e^{j\omega_0 t}$, the phase of the output will exceed, or lead, the phase of the input. Similarly, an LTI system is said to have phase lag at a particular frequency $\omega = \omega_0$ if, for the input $e^{j\omega_0 t}$, the phase of the output will lag the phase of the input. Consider two systems with the following frequency responses:

$$\text{(i) } H_1(j\omega) = \frac{1 + j(\omega/5)}{1 + j(5\omega)} \qquad \text{(ii) } H_2(j\omega) = \frac{1 + j(5\omega)}{1 + j(\omega/5)}.$$

Which has phase lead at any positive frequencies?

三、(8%)

There is a continuous-time periodic signal $x(t)$ with fundamental period $T$ and its one-period signal is $\hat{x}(t)$ (i.e., $\hat{x}(t)$ is only with one cycle of $x(t)$ and aperiodic). Below is the magnitude spectrum of $\hat{x}(t)$ where $\omega_M = \dfrac{4\pi}{T}$.

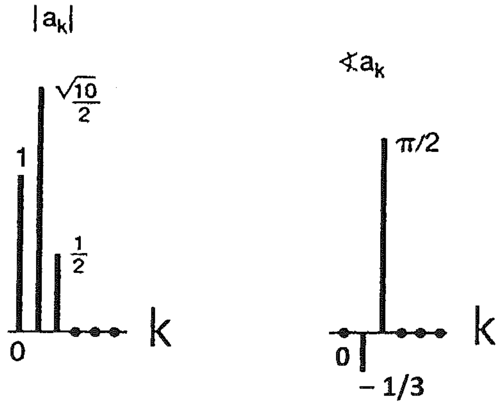



Plot the magnitude of Fourier series coefficients of $x(t)$.

四、(7%)

Given a discrete-time periodic signal x[n], below are the plots **(see NEXT page)** of half (i.e., the positive frequency part or positive k part) of the magnitude (left plot) and phase (right plot) of its Fourier series coefficients within one fundamental period N (N=10) in frequency domain. x[n] is real. Find x[n].

注意:背面有試題




五、(15%)

Given a zero-mean Gaussian pulse defined as follows.

$$x(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-t^2}{2\sigma^2}}.$$

(一) (8%) Find $X(0)$ ($X(j\omega)$ is the Fourier transform of $x(t)$).

(二) (7%) Find its Fourier transform spectrum $X(j\omega)$.

(Hint: $\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\pi}$.)

六、(5%)

Determine whether the following signals are periodic, and for those are, find the fundamental period.

(一) (3%) $x(t) = 2\cos\left(\frac{2\pi t}{3}\right) + 3\cos\frac{2\pi t}{7}$

(二) (2%) $x[n] = \cos^2(\frac{\pi}{4}n)$

七、(9%)

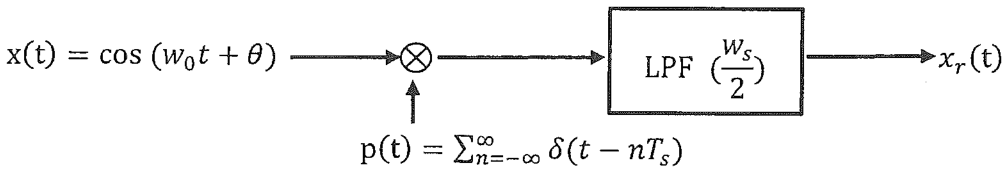
Consider the sampling system depicted in the flowing figure **(see NEXT page)**, suppose $x(t) = \cos(w_0 t + \theta)$ and $w_s = \frac{2\pi}{Ts}$. LPF($\frac{w_s}{2}$) denotes the ideal rectangular low-pass filter with a unit gain and a cut-off frequency of $\frac{w_s}{2}$. Determine the output signal $x_r(t)$ at the conditions of different sampling rate.

(一) (3%) $w_0 = \frac{w_s}{3}$

(二) (3%) $w_0 = \frac{3\,w_s}{4}$

(三) (3%) $w_0 = \frac{7\,w_s}{6}$

注意:背面有試題




八、(6%)

For the signal $x(t) = e^{-6t}u(t) * \frac{\sin(Wt)}{\pi t}$, sampled with sampling interval $T_s$ = ?, which guarantee that there will be no aliasing. * denotes the operator taking convolution.

九、(10%)

A system has the indicated transfer function $H(s) = \frac{2s^2+2s-2}{s^2-1}$. Determine the system impulse response, assuming that

(一) (4%) the system is causal.
(二) (4%) the system is stable.
(三) (2%) Can the system be both causal and stable? (Answer YES or NO)

十、(15%)

Consider the below causal LTI system:

(一) (7%) Find the transfer function (i.e., H(z)).
(二) (5%) Find the impulse response (i.e., h[n]).
(三) (3%) Discuss the stability of the system.