

所有題目皆須附計算過程 無計算過程者不予計分

1. (28%) Consider a unity feedback control system as given in Figure 1 below, where G_p and G_c denote the system plant and controller, respectively. Let the characteristic equation of the closed-loop system be given by $s^3 + 2s^2 + (a+1)s + 3Ks + 6K = 0$.
 - (a) (8%) Let $G_c = K$. Then find the function $G_p(s)$ and plot the root loci of the closed-loop system with $a = 0$ for $K \geq 0$.
 - (b) (4%) Solve the steady-state error for input $U(s) = \frac{1}{s}$ and $U(s) = \frac{1}{s^2}$, respectively, with $a = 0$ and $K = 1$.
 - (c) (8%) Find the value of a and the corresponding value of K so that the closed-loop system will have a triple pole on the real axis for $K \geq 0$.
 - (d) (8%) Find the range of a with $a \geq 0$ so that the closed-loop system will have only one breakaway and/or break-in points on the negative real axis for $K \geq 0$.

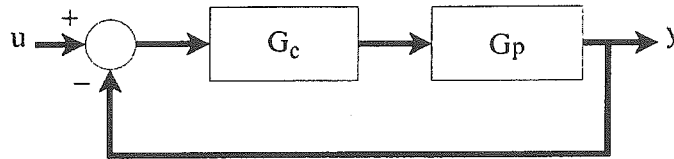


Figure 1: Unity feedback control system for Problem 1 and 2

2. (22%) Consider the closed-loop feedback system as given in Figure 1 above.
 - (a) (4%) Let $G_p = \frac{s(s+2)}{s^3 + 2s^2 + 10}$ and $G_c = K$. Find the range of K for guaranteeing the asymptotic stability of the system.
 - (b) (6%) Let $G_p = \frac{68}{s(s+4)}$ and $G_c = 1$. Solve for the maximum overshoot of the system output $y(t)$ and the corresponding time for the unit-step input, i.e., $u(s) = \frac{1}{s}$.
 - (c) (6%) Let $G_p = \frac{68}{s(s+4)}$ and $G_c = K$. Find the value of K so that the closed-loop poles will have damping ratio $\zeta = 0.5$. In addition, what will be the value of the corresponding undamped natural frequency ω_n ?
 - (d) (6%) Let $G_p = \frac{s+1}{s(s+4)}$. Design a controller G_c so that the closed-loop poles will have undamped natural frequency $\omega_n = 10$. In addition, what will be the value of the corresponding damping ratio ζ ?

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3. (26%) Consider the feedback control system in Figure 2. Let $C(s) = K > 0$. Suppose that $G(s)$ is stable, and the Nyquist plot of $KG(s)$ for $K = 1$ is shown in Figure 3 (Note: the Nyquist path is in the clockwise direction). To answer the following questions, assume $K = 1$ for questions (a)-(d).
- (a) (4%) If r is the unit step function, what is $\lim_{t \rightarrow \infty} e(t)$?
- (b) (4%) If r is the impulse function, show that $y(0) \neq 0$.
- (c) (4%) If $r = \cos(0.53t)$, what is the steady-state output $y(t)$?
- (d) (6%) Let $S(s)$ be the sensitivity function of the system, i.e. the closed-loop transfer function from r to e . Find a reasonable upper bound of $\max_{-\infty < \omega < \infty} |S(j\omega)|$.
- (e) (8%) Find all ranges of positive K such that the closed-loop system is stable.

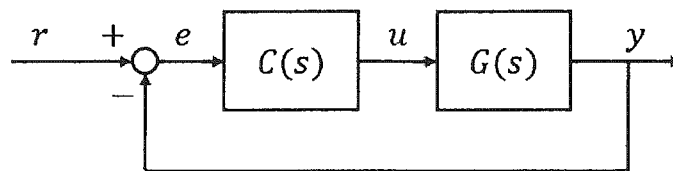


Figure 2: Feedback control system in Problem 3 and 4

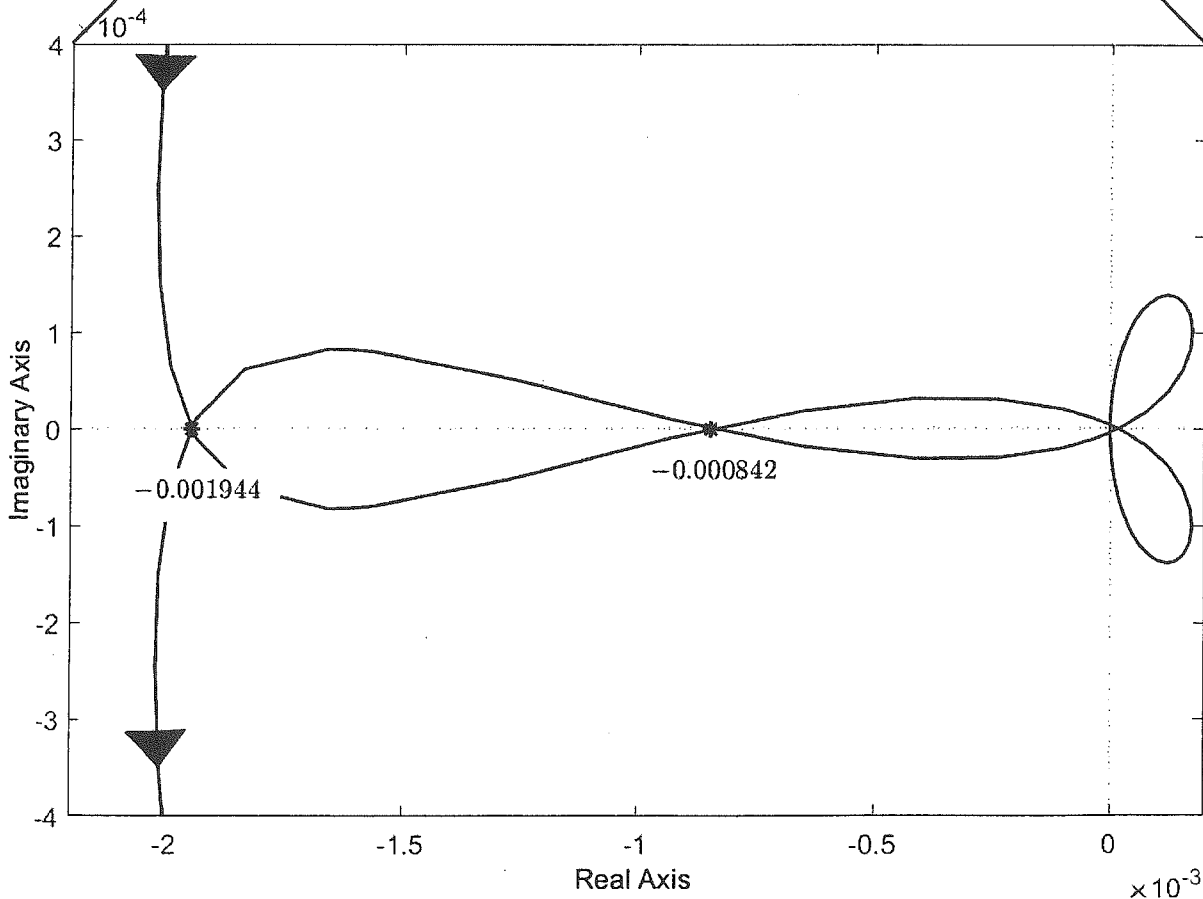
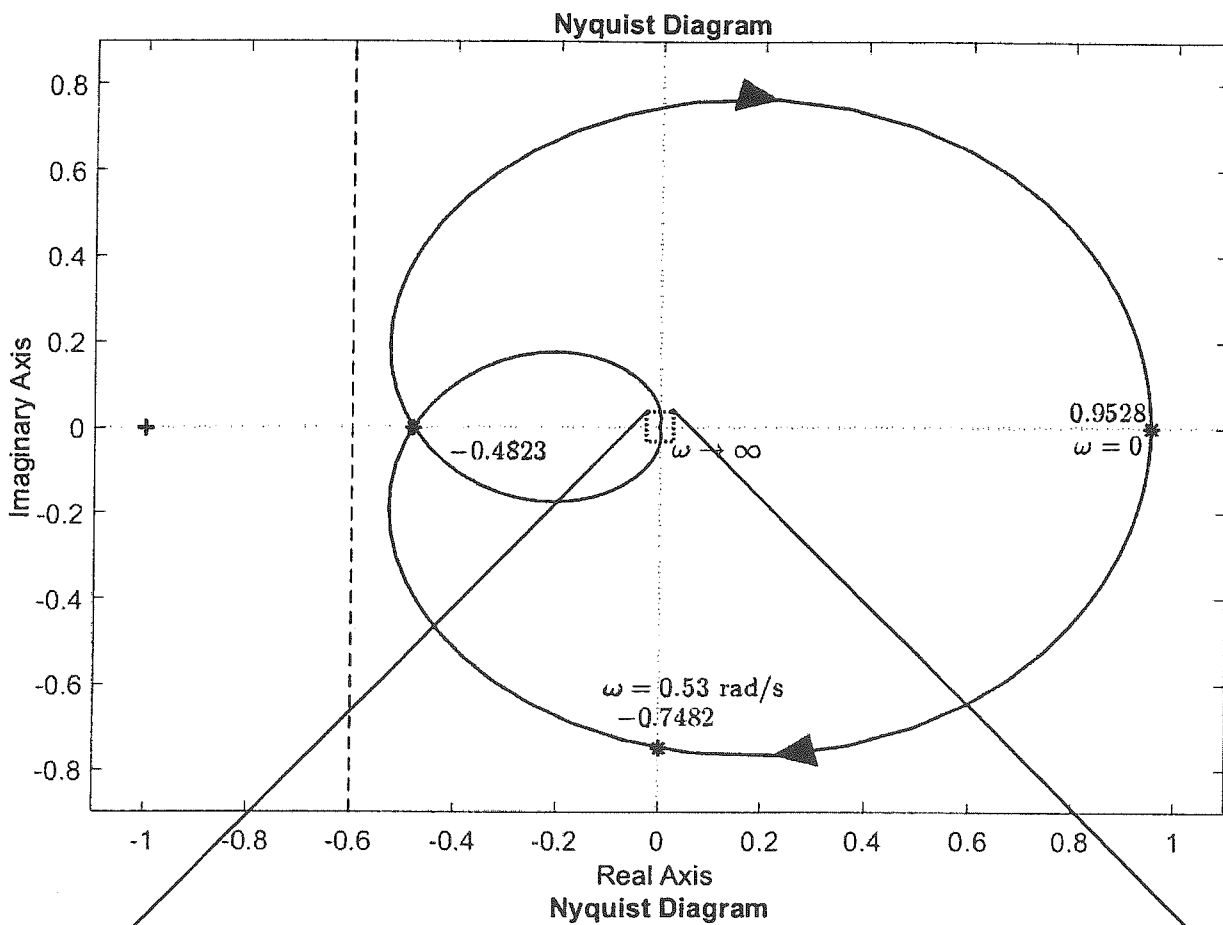


Figure 3: (upper) Nyquist plot of $KG(s)$; (lower) enlargement around the origin.

4. (24%) Consider the feedback control system in Figure 2. The Bode plot of $G(s)$ is given in Figure 4. Let $M(s)$ be the closed-loop transfer function from r to y , and $M(s)$ is stable when $C(s) = 1$. Answer the following questions.
- (a) (4%) Assume $C(s) = 1$. Let ω_p be the phase crossover frequency. What are $M(0)$ and $M(j\omega_p)$?
- (b) (4%) Assume $C(s) = 1$. Based on the information in Figure 4, argue that the resonant peak of $M(s)$ is at least 9.75 dB.
- (c) (6%) Sketch the Nyquist plot for $C(s) = 1$. Suppose that the Nyquist path is clockwise and detours to the right when there is any pole of $G(s)$ on the imaginary axis. You should indicate the intersection of the Nyquist plot and the real axis, and the angles for $\omega \rightarrow 0^+$ and $\omega \rightarrow \infty$.
- (d) (4%) Let $C(s) = K > 0$. Find the value of K such that the step response of the closed-loop system has sustained oscillation (i.e. oscillation with identical magnitude). What is the frequency of this sustained oscillation?
- (e) (6%) Let $C(s) = K_p + \frac{K_I}{s}$ be a PI controller. Find K_p and K_I such that the phase margin of the system is more than 60 degrees.

Remark: State your procedure for tuning K_p and K_I . You do not have to verify your design. Just mention how you will tune K_p and K_I if the desired phase margin is not achieved.

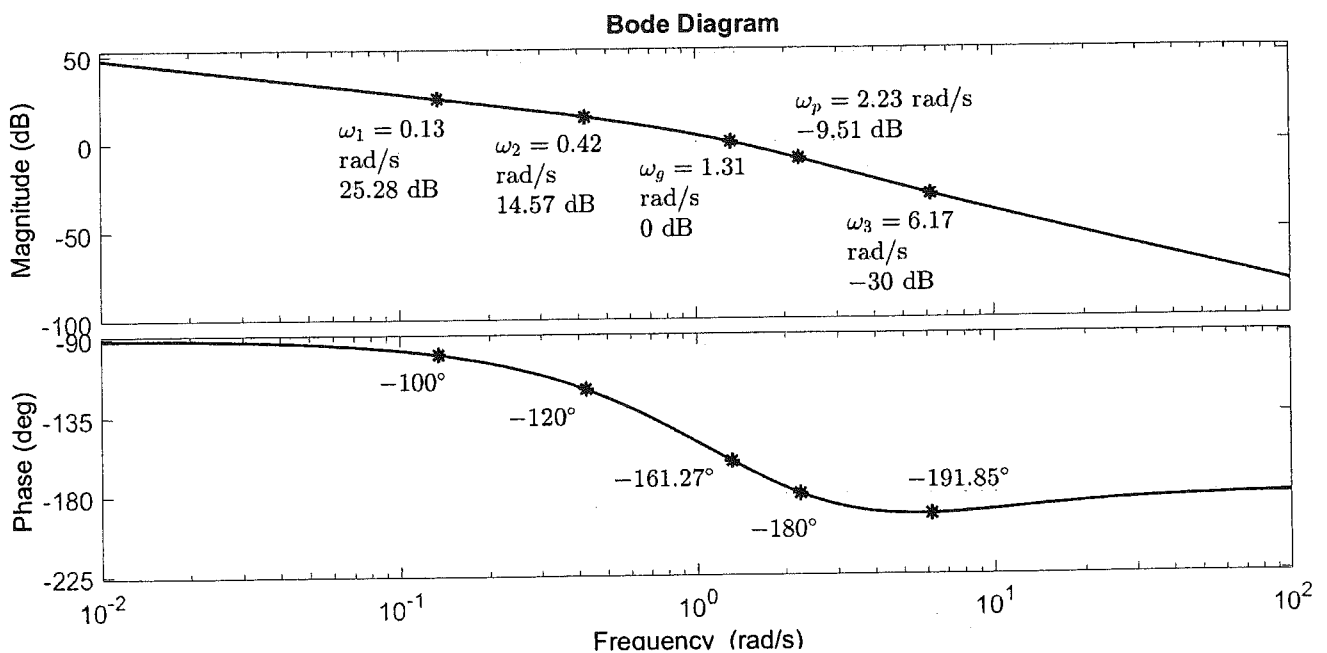


Figure 4: Bode Plot of $G(s)$ for Problem 4

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