計算題應詳列計算過程，無計算過程者不予計分

1. (24%) Let $G(s) = \frac{1}{P(s)}$ be the transfer function of a linear time-invariant system, where

$$P(s) = s^4 + (1 - k_1 - k_2)s^3 + 2k_1 s^2 + (2k_1 - 1)(1 - k_1 - k_2)s + 2k_1 - 1$$

and $k_1, k_2 > 0$. Find all conditions on k_1 and k_2 for each of following cases. If no k_1 and k_2 can meet the case, explain the reason.

   (a) (6%) $G(s)$ is stable.

   (b) (6%) $G(s)$ has exactly one pair of pure imaginary poles.

   (c) (6%) $G(s)$ has four poles on the imaginary axis.

   (d) (6%) $G(s)$ has one pole on the left-half plane and three poles on the right half plane. (Note: the imaginary axis is neither a part of the left-half plane, nor a part of the right-half plane.)

2. (13%) Consider a second order system with a transfer function $G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$, where $\xi > 0$ and $\omega_n > 0$ are the damping ratio and the undamped natural frequency of $G(s)$, respectively. Let $y(t)$ be the unit-step response of $G(s)$. Suppose that y_1 is the maximum overshoot, which takes place at time t_1, while $1 - y_2$ is the first local minimum of $y(t)$ for $t > t_1$ (see Figure 1). Suppose $\frac{y_1}{y_2} = 2.6858$ and $t_1 = 6.5866$ sec.

   (a) (8%) Find the values of ξ and ω_n.

   (b) (5%) Consider the unity feedback system in Figure 2, where $K > 0$. Let r in Figure 2 be the unit-step input. Find the range of K such that the percent maximum overshoot of y is less than 50% and the steady-state error is less than 0.6.

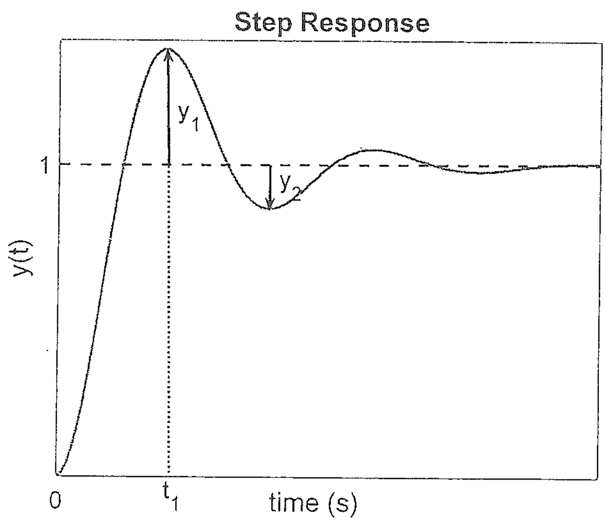

Figure 1: The unit-step response of Problem 2

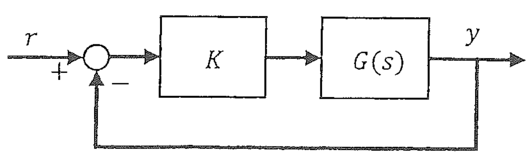

Figure 2: The unity feedback system

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3. (13%) Consider the unity feedback control system in Figure 2, where $K > 0$ and

$$G(s) = \frac{s^2 - s + 4.25}{s(s^2 - 0.8s + 13.12)}$$

(a) (8%) Draw the root locus of the system for $K > 0$. Find the arrival angles of the zeros, the departure angles of the complex poles, and the intersections of the root locus with the imaginary axis, if there is any.

(b) (5%) Find the range of K for the closed-loop system to be stable.

4. (38%) Consider a unity-feedback system with the open-loop transfer function $G(s) = \dfrac{K(s+1)}{s(s-1)}$, where $G(j\omega) = \dfrac{-2K}{1+\omega^2} + j\dfrac{K(1-\omega^2)}{\omega(1+\omega^2)}$.

(a) (10%) Show that part of the root locus is a circle. Write down the circle equation by letting $s = x + jy$.

(b) (10%) Sketch the Nyquist plot for K>0 as detail as possible.

(c) (4%) Analyze the stability from the Nyquist plot for all K.

(d) (3%) When K=1/3 and $r(t) = 1$, what is the steady state output?

(e) (3%) When K=3 and $r(t) = \cos(t)$, what is the steady state output?

(f) (8%) If the gain crossover frequency=$\sqrt{3}$ rad/sec, what are the corresponding K, the Gain Margin, Phase Margin and the phase crossover frequency?

5. (12%) Given the system $\dddot{y} + 4\ddot{y} + 5\dot{y} + 2y = u$. Define the states $x_1 \triangleq y$, $x_2 \triangleq \dot{y}$, and $x_3 \triangleq \ddot{y}$.

(a) (5%) Sketch the state equation description with Controllability Canonical Form.

(b) (4%) If all states can be measured, design the state feedback controller, $u = -Kx + r$ such that the desired closed-loop poles of the controller locates at $-2 \pm j2$ and -10.

(c) (3%) If only $y(t)$ can be measured, an observer design is necessary by output feedback. Design the observer, so that the closed-loop poles of the observer locates at $-10 \pm j10$ and -20.

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