1. (20 pts.) A solid object of mass M is put on two identical hollow cylinders (each with mass m and radius r). The mass M is connected horizontally to two vertical walls by two identical springs with spring constant k. Only pure rolling occurs between mass M, two hollow cylinders and the ground. Assume the oscillation amplitude is small. (a) Find the rotational inertia of a cylinder about its central axis, (b) Find the oscillation frequency of M if the mass m of each hollow cylinder is negligible, and (c) Find the oscillation frequency of the system with the finite m included.

2. (20 pts.) A meteorite, of mass 1000kg, and relative speed 50m/sec comes from far apart and approaches the Jupiter. Depends on the impact parameter the Jupiter may or may not collide with the meteorite. At what impact parameter the meteorite will just touch the surface of the Jupeter? (You can assume that the Jupiter is fixed in the space.) The mass and the radius of Jupiter are 1.9×10^{27}kg and 7.0×10^{7}m respectively. The gravitational constant is $6.67 \times 10^{-11} \mathrm{N \cdot m^2/kg^2}$.

3. (10 pts.) A traveling wave on a string is described by

$$y = 2.0 \sin[2\pi(t/0.4 + x/80)]$$

where x and y are in centimeters and t is in seconds. (a) For $t = 0$, plot y as a function of x for $0 \leq x \leq 160$ cm. (b) Repeat (a) for $t = 0.1$s. (c) From your graphs, what is the wave speed, and in which direction ($+x$ or $-x$) is the wave traveling?

4. (10 pts.)

   (a) The latent heat of fusion of the water is 3.3×10^5 J/kg. Calculate the entropy difference between one mole liquid water and ice at 0 °C.

   (b) At the lobby of a hotel, there is a large window. The area of the window glass is 2m×1m, and the thickness of the glass is 1cm. At a cold winter day, the temperature outdoor is -10 °C. Inside the hotel, the temperature is 20 °C. Given that the thermal conductivity of the glass is 0.8 W/m ·°C, calculate the heat flux throught the window (by heat conduction alone).

5. (20 pts.) A straight, infinitely-long flat conductor (with zero thickness) of width $2a$ carries a current I. The coordinate axes are chosen so that the edges of the conductor are situated at $x = \pm a$, and the current flows in the direction of the positive z-axis. (So that the conductor is on the x–z plane.)

   (a) Find the magnetic field $\mathbf{B}$ at the location $(x_0, 0, 0)$.

   (b) Find the magnetic field $\mathbf{B}$ at the location $(0, y_0, 0)$.

   (c) Show that for a general position $(x_0, y_0, 0)$,

$$B_x = -\frac{\mu_0 I}{4\pi a}\alpha, \qquad B_y = \frac{\mu_0 I}{4\pi a}\ln\frac{r_2}{r_1}$$

   Here r_1 is the distance between $(x_0, y_0, 0)$ and $(a, 0, 0)$, and r_2 the distance between $(x_0, y_0, 0)$ and $(-a, 0, 0)$. The angle α is that between r_1 and r_2.

6. (20 pts.) Consider a RLC circuit consists of a DC voltage source V_0, a resistor R, and a capacitor C. Assume that at $t = 0$ the capacitor is uncharged and the circuit is connected at this instant.

   (a) Make a graph showing the current $I(t)$ of the circuit and the voltage across the capacitor $V_c(t)$ as functions of time. Mark the value of RC on your graph.

   (b) Now if the voltage source is a square wave $V(t)$ such that

$$\begin{aligned} V(0 < t < a) &= V_0 \\ V(a < t < 2a) &= 0 \\ V(2a < t < 3a) &= V_0 \\ V(3a < t < 4a) &= 0 \\ &\vdots \end{aligned}$$

   Make a graph showing $V(t)$, $I(t)$, and $V_c(t)$ as functions of time.

   (c) Now consider a different circuit consists of only a capacitor C and a inductor L. Assume that at $t = 0$ the current I is zero and there is voltage V_0 across the capacitor. Write down a differential equation for the current $I(t)$ and solve it.