

國立中央大學八十九學年度轉學生入學試題卷

I. 資電 北科學院 二年級

科目：微積分

共一頁 第一頁

參考用

1 (10 points) Evaluate the given integrals.

(a) $\int \frac{\sin x}{1+\sin^2 x} dx.$

(b) $\int \cos(\ln x) dx.$

2 (10 points) Perform the following differentiations.

(a) $\frac{d}{dx} \int_{x^2}^1 (t - \sin^2 t) dt.$

(b) $\frac{d}{dx} [x^{(2^x)}].$

3 (10 points) (a) Find the area of the region that consists of all points that lie within the circle $r = 2 \cos \theta$ but outside the circle $r = 1$.

(b) Find the points of the cardioid $r = 1 - \cos \theta$ at which the tangent line is vertical.

4 (10 points) Find the indicated limits.

(a) $\lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x \sin\left(\frac{1}{t+1}\right) dt.$

(b) Find the limit of the sequence: $\lim_{n \rightarrow \infty} (\sqrt[n]{n} - 1)^{\frac{1}{\sqrt[n]{n}}}$.

5 (10 points) Given an ϵ, δ proof that

$$\lim_{x \rightarrow c} \sqrt{x} = \sqrt{c} \text{ for } c > 0.$$

6 (10 points) Find the extreme values and the saddle points (if any) of

$$f(x, y) = x^4 - 2x^2 + y^2 - 2.$$

7 (10 points) Expand the following functions f in powers of x

(a) $f(x) = \ln(1 - x^2), \quad x \in (-1, 1).$

(b) $f(x) = \int_0^x \frac{1 - \cos t}{t^2} dt.$

8 (10 points) Evaluate

$$\int_C x^2 y dx + xy dy,$$

where C is the circular path $y = \sqrt{1 - x^2}$ connecting $(1, 0)$ to $(0, 1)$.

9 (10 points) Find the volume of the solid bounded by the circular cylinder

$$x^2 + y^2 = 1, \text{ the plane } z = 0, \text{ and the plane } x + z = 1.$$

10 (10 points) (a) Show that the polynomial function $p(x) = 2x^3 + 5x - 1$ has exactly one real zero.

(b) Prove that for all real x and y

$$|\cos x - \cos y| \leq |x - y|.$$