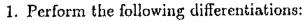
國立中央大學八十五學年度轉學生入學試題卷

科目:微積分



(a) (5%) Let
$$F(x) = \int_1^{x^2} e^{-1/t^2} dt$$
 for $x \neq 0$. Find $F'(x)$.

(b) (15%) Let

$$f(x) = \begin{cases} c^{-1/x^2} & \text{if } x \neq 0\\ 0 & \text{if } x = 0 \end{cases}$$

Find f'(0) and f''(0). (Give the details of computations).

2. (15%) Compute the integral

$$\int \frac{dx}{x^4 + 2x^2 - 3}.$$

- 3. (15%) Find the volume of the solid bounded by the surface $x^2 + y^2 + z = 1$ and the coordinate xy plane.
- 4. (10%) Let $f(x,y) = x^2 2xy + 3y^2 + x 5y + 1$. Find the extremum value or the saddle point of f, if any.
- 5. (10%) Let f(t) = (f₁(t), f₂(t), f₃(t)) be a vector-valued function for t ∈ [a, b]. If f is continuous on [a, b] and is differentiable on (a, b), prove that there is ξ ∈ (a, b) such that the tangent vector of f at ξ is orthogonal to the vector f(a) × f(b).
- 6. Consider the power series $\sum_{n=1}^{\infty} (n+1)x^n$.
 - (a) (5%) Find the radius R of convergence of this series.
 - (b) (15%) Find the sum function f(x) of this series for |x| < R.
- 7. (10%) Let $\{a_n\}_{n=1}^{\infty}$ be a decreasing sequence of positive terms with $\lim_{n\to\infty} a_n = 0$. Let f be the function defined for $x \in [0, \infty)$ by

$$f(x) = (-1)^{n-1}a_n$$
 for $x \in [n-1, n), n = 1, 2, 3, ...$

Prove that the improper integral $\int_0^\infty f(x) dx$ converges.

