

一、填充題：共8題，每題8分、總計64分。請在答案卷上列出題號並依序作答。

請注意：本部分共8題，命題型態為填充題，請依題號順序獨立列出，勿同時陳列出計算過程，倘若答案被包含在演算過程內，將被視為試算流程，不另行挑出計分。

(1) Evaluate the integral:

$$\int_0^{\infty} \int_y^{\infty} y^2 e^{-x^2} dx dy.$$

(2) Find the inflection points of the function

$$f(x) = e^x \sin x$$

over the interval $[0, 2\pi]$. Remind you that the inflection point is where concavity changes and it is a point on the graph.

(3) Evaluate

$$\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}.$$

(4) Find the interval of convergence of the following power series:

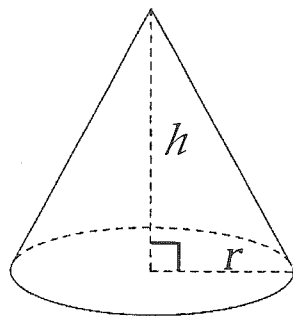
$$\sum_{n=0}^{\infty} \frac{(-1)^n (n+2)}{3^n} (x-3)^n.$$

(5) Let

$$f(x) = \begin{cases} \sin(2x), & \text{if } x \leq 0; \\ 2x^2 + ax + b - 3, & \text{if } x > 0. \end{cases}$$

Find a and b such that f is differentiable at 0. Write your answer as (a, b) .

(6) Find the volume of the right circular cone of base radius r and height h as shown in the following figure. Write your answer in terms of h and r .



(7) Find all the extrema of the function $f(x, y) = x^3 - 12xy + 8y^3$ on $(-\infty, \infty) \times (-\infty, \infty)$.

(8) Suppose that $x^2 + xy = \sin x$. Find dy/dx in terms of x and y .

科目 微積分 類組別 A2 共 2 頁 第 2 頁

二、計算、證明題：共3題，每題12分、總計36分。請將題號標明清楚。

請注意：須詳細寫出計算及證明過程，否則不予計分。請書寫清楚乾淨，過程解釋清楚，任何書寫或是解釋不清將被扣分。

(1) Determine for which $p \geq 0$ the series

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^p}$$

converges or diverges.

(2) A tank initially contains 20 gallons of pure water. Brine (high-concentration solution of salt) containing 2 pounds of salt per gallon flows into the tank at a rate of 4 gallons per minute, and the well-stirred mixture flows out of the tank at the same rate. How much salt is present at the end of 10 minutes?

(3) Find the area of the region R that is completely enclosed by the graphs of the functions

$$y = f(x) = e^x - e^{-x} + 3 \quad \text{and} \quad y = g(x) = 2e^x + 5e^{-x} - 2.$$