

\*請在答案卷內作答

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上。

1. Find  $\frac{d^{20}}{dx^{20}} \left[ \left( \sin \frac{x}{2} \right) \left( \cos \frac{x}{2} \right) \right]$ . Answer : \_\_\_\_\_

2. If  $f(x) = \ln x + \tan^{-1} x$ , find  $(f^{-1})'(\frac{\pi}{4})$ . Answer : \_\_\_\_\_

3. Evaluate  $\int_0^1 \frac{xe^{2x}}{(2x+1)^2} dx$ . Answer : \_\_\_\_\_

4. Find the interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{2n(x-3)^n}{(n+1)!}$ . Answer : \_\_\_\_\_

5. Find the limit:  $\lim_{x \rightarrow \infty} \frac{1}{x \ln x} \int_1^x \ln t dt$ . Answer : \_\_\_\_\_

6. Find the maximum rate of change of  $f(x, y) = x^2y + e^{xy} \sin y$  at  $(1, 0)$ . Answer : \_\_\_\_\_

7. Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{\cos(x^2 + y^2) - 1}{x^2 + y^2}$ .

Answer : \_\_\_\_\_

8. Evaluate  $\int_0^1 \int_0^{1-x} \sqrt{x+y}(y-2x)^2 dy dx$  by applying the transformation  $u = x+y, v = y-2x$  and integrating over an appropriate region in the  $uv$ -plane. Answer : \_\_\_\_\_

參考用

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Sketch the region of integration, reverse the order of integration, and evaluate the integral  $\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dy dx$ .

2 Find the counterclockwise circulation for the field  $F = (x + e^x \sin y)\mathbf{i} + (x + e^x \cos y)\mathbf{j}$  along the curve  $C$  : The right-hand loop of the lemniscate  $r^2 = \cos 2\theta$ .

3. A manufacturer finds that it takes  $x$  units of labor and  $y$  units of capital to produce  $f(x, y) = 100x^{3/4}y^{1/4}$  units of product. If a unit of labor costs \$100, a unit of capital costs \$200, and \$200,000 is budgeted for production, determine how many units should be expended on labor and how many units should be expended on capital in order to maximize production.