1. (10 points): Let

\[ I = \int_C \frac{y}{x^2 + y^2} \, dx - \frac{x}{x^2 + y^2} \, dy \]

where \( C \) is a circle oriented counterclockwise.

(a) Evaluate \( I \) if \( C \) is given by \((x - 2016)^2 + (y - 2016)^2 = 1\).

(b) Evaluate \( I \) if \( C \) is given by \( x^2 + y^2 = 1 \).

2. (10 points): Find the maximum and minimum values of the function \( f(x, y, z) = x^2 - y^2 \) on the surface \( x^2 + 2y^2 + 3z^2 = 1 \).

3. (10 points): Compute

\[ \lim_{x \to +\infty} \left( \sqrt{x + \sqrt{x + \sqrt{x - \sqrt{x}}} - \sqrt{x}} \right) \]

4. (10 points): For what positive \( x \) does the following series converge?

\[ \sum_{n=1}^{\infty} (\sqrt{x} - 1) \]

5. (10 points): Let \( B = \{ (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1 \} \). Evaluate the integral

\[ \iiint_B \frac{x^4 + 2y^4}{x^4 + 4y^4 + z^4} \, dV. \]

6. (10 points): The \( n \)-th derivative of \( \frac{1}{x^{2015} - 1} \) has the form \( \frac{P_n(x)}{(x^{2015} - 1)^{n+1}} \) where \( P_n(x) \) is a polynomial.

Find \( P_n(1) \) for all \( n \geq 0 \).

7. (20 points): (a) Prove that

\[ \int_0^\infty \left( \frac{\sin x}{x} \right)^2 \, dx = \int_0^\infty \frac{\sin x}{x} \, dx. \]

(b) Evaluate the improper integral

\[ \int_0^\infty \frac{\sin x}{x} \, dx. \]

8. (10 points): For each continuous function \( f : [0, 1] \to \mathbb{R} \), let \( I(f) = \int_0^1 x f(x)(x - f(x)) \, dx \). Find the maximum value of \( I(f) \) over all such functions \( f \).

9. (10 points): Evaluate

\[ \int_0^\infty \left( x - \frac{x^3}{2} + \frac{x^5}{2 \cdot 4} - \frac{x^7}{2 \cdot 4 \cdot 6} + \cdots \right) \left( 1 + \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} + \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots \right) \, dx. \]