科目 微積分 類組別 A3 A4 A6 共 1 頁 第 1 頁

＊請在答案卷內作答

1. (10 points): Let

$$I = \int_C \frac{y}{x^2+y^2}\,dx - \frac{x}{x^2+y^2}\,dy$$

where C is a circle oriented counterclockwise.

(a) Evaluate I if C is given by $(x-2016)^2+(y-2016)^2=1$.

(b) Evaluate I if C is given by $x^2+y^2=1$.

2. (10 points): Find the maximum and minimum values of the function $f(x,y,z)=x^2-y^2$ on the surface $x^2+2y^2+3z^2=1$.

3. (10 points): Compute

$$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$$

4. (10 points): For what positive x does the following series converge?

$$\sum_{n=1}^{\infty}\left(\sqrt[n]{x}-1\right)$$

5. (10 points): Let $B=\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2\le 1\}$. Evaluate the integral

$$\iiint_B \frac{x^4+2y^4}{x^4+4y^4+z^4}\,dV.$$

參考用

6. (10 points): The n-th derivative of $\frac{1}{x^{2016}-1}$ has the form $\frac{P_n(x)}{(x^{2016}-1)^{n+1}}$ where $P_n(x)$ is a polynomial. Find $P_n(1)$ for all $n\ge 0$.

7. (20 points): (a) Prove that

$$\int_0^{\infty}\left(\frac{\sin x}{x}\right)^2 dx = \int_0^{\infty}\frac{\sin x}{x}\,dx.$$

(b) Evaluate the improper integral

$$\int_0^{\infty}\frac{\sin x}{x}\,dx.$$

8. (10 points): For each continuous function $f:[0,1]\to\mathbb{R}$, let $I(f)=\int_0^1 xf(x)(x-f(x))\,dx$. Find the maximum value of $I(f)$ over all such functions f.

9. (10 points): Evaluate

$$\int_0^{\infty}\left(x-\frac{x^3}{2}+\frac{x^5}{2\cdot 4}-\frac{x^7}{2\cdot 4\cdot 6}+\cdots\right)\left(1+\frac{x^2}{2^2}+\frac{x^4}{2^2\cdot 4^2}+\frac{x^6}{2^2\cdot 4^2\cdot 6^2}+\cdots\right)dx.$$