科目　微積分　　　類組別　　A3 A4 A7　　　　　　　共 / 頁 第 / 頁

＊請在答案卷內作答

甲、填充題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲、）部分，共 8 題，命題型態為填充題，不必詳列計算過程，倘若答案被包含在演算過程，將被視為試算流程，無法計分。

1. How many critical points does the function $f(x) = |x^2 - 1|$ have? Answer :＿＿＿＿

2. If $\lim\limits_{x \to 0} \dfrac{\sqrt{ax + b} - 2}{x} = 1$, then $a - b =$? Answer :＿＿＿＿

3. Suppose f'' is continuous on $[0,1]$, $f(1) = 2$, $f'(1) = 2$ and the average value of f on $[0,1]$ is 2. Evaluate $\displaystyle\int_0^1 x^2 f''(x)\,dx$. Answer :＿＿＿＿

4. Find $g'(-\frac{1}{2})$, where $g(x)$ is the inverse of $f(x) = \dfrac{x^3}{x^2 + 1}$. Answer :＿＿＿＿

5. Find the directional derivative of $f(x,y) = \tan^{-1}(\frac{x}{y})$ at $P_0(1,1)$ in the direction of $\mathbf{v} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$. Answer :＿＿＿＿

6. Evaluate $\displaystyle\int_0^6 \int_{x/3}^2 x\sqrt{y^3 + 1}\,dy\,dx$. Answer :＿＿＿＿

7. Let R be the solid inside $x^2 + y^2 + z^2 = 1$, outside $z = \sqrt{x^2 + y^2}$ and above the xy-plane. Find the volume of R. Answer :＿＿＿＿

8. Evaluate the line integral $\displaystyle\int_C 2xy\,dx + (x^2 + 2x)\,dy$, where C : boundary of the region lying between the graphs of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the circle $x^2 + y^2 = 1$.
Answer :＿＿＿＿

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1 Find any extrema of the function $f(x,y) = e^{-xy/4}$ subject to the constraint $x^2 + y^2 \le 1$.

2. For the function
$$f(x,y) = \begin{cases} \dfrac{-5xy}{x^2 + y^2}, & \text{if } (x,y) \ne (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$
show that $f_x(0,0)$ and $f_y(0,0)$ both exist, but that f is not differentiable at $(0,0)$.

3. Determine whether the series $\displaystyle\sum_{n=3}^{\infty} \dfrac{\ln n}{\ln \ln n}$ converges absolutely or conditionally, or diverges.