

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. If f is a continuous function such that $\int_0^x f(t) dt = xe^{2x} + \int_0^x e^{-t} f(t) dt$ for all x , find an explicit formula for $f(x)$. Answer : _____

2. In what direction is the derivative of $f(x, y) = \frac{(x^2 - y^2)}{(x^2 + y^2)}$ at $P(1, 1)$ equal to zero? Answer : _____

3. Find the maximum value of $x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$. Answer : _____

4. Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x . How large can $f(2)$ possibly be? Answer : _____

5. Find the tangent plane of the surface $\cos \pi x - x^2 y + e^{xz} + yz = 4$ at the point $P_0(0, 1, 2)$. Answer : _____

6. Evaluate $\iint_R \frac{x-2y}{3x-y} dA$, where R is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$, and $3x-y=8$. Answer : _____

7. Find the area of the surface cut from the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 2$. Answer : _____

8. Evaluate the integral $\oint_C (6y+x) dx + (y+2x) dy$ along the circle $C : (x-2)^2 + (y-3)^2 = 4$. Answer: _____

乙、計算、證明題：共 3 大題，每大題 12 分，共 36 分。需詳細寫出演算過程、否則不予計分。

1. Evaluate the following limits.

(a) $\lim_{x \rightarrow (\pi/4)^-} (\tan x)^{\tan(2x)}$. (b) $\lim_{n \rightarrow \infty} \left(\sum_{j=1}^n \frac{\sqrt{n^2 - j^2}}{n^2} \right)$.

2. (a) Test the series $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$ for convergence or divergence.

(b) Let $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$. Find the intervals of convergence for f' and f'' .

3. Evaluate

(a) $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2 - y^2}} e^{x^2 + y^2} dx dy$. (b) $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{dy dx}{y^4 + 1}$.